Time-delay Tolerant Control of an Omnidirectional Multi-agent System for Transport Operations

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Outline

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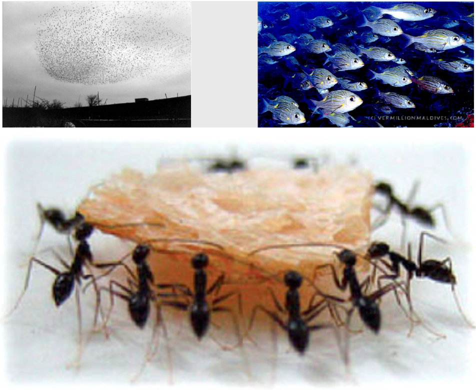
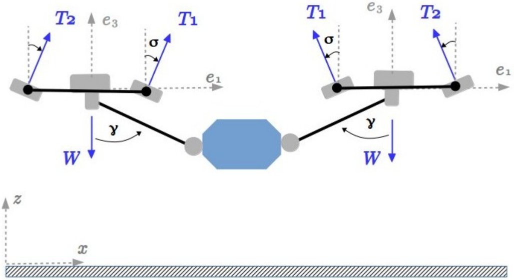
- The problem of synchronization of multiple agents arises in numerous applications, both in natural and in man-made systems.
- Examples from nature include:

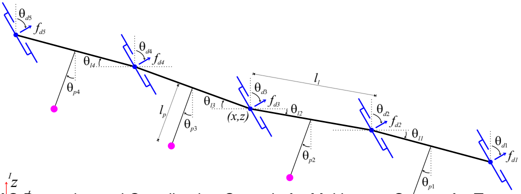


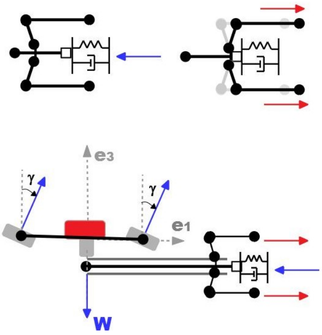
Bird flock, fish school and collective transport

Introduction

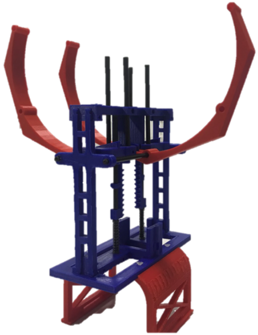
Context and Main Goal





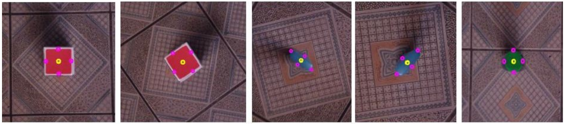


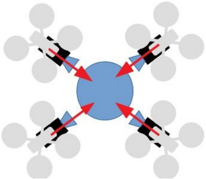
e3
γ
γ
e1
W

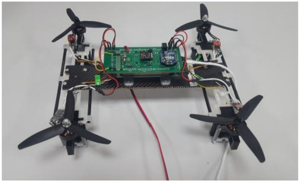


During grasping or manipulation operations, contacts between the end-effectors and the object depend on different aspects, such as: friction, vector force direction and magnitude.

A reliable contact point between the gripper and the object is considered as optimal if the grabbing force is orthogonal to the object's surface.







- Time-delay Control of an Omnidirectional Multi-agent System for Transport Operations.
  J. U. Alvarez-Muñoz, J. Escareno, F. Méndez-Barrios, I. Boussaada and S. I. Niculescu
  22nd IEEE International Conference on System Theory, Control and Computing, Sinaia, Rumania (presented yesterday!)

- Time-delay-based PID Control of an Omnidirectional Multi-agent System for Transport Operations
  J. U. Alvarez-Muñoz, J. Escareno, F. Méndez-Barrios, I. Boussaada and S. I. Niculescu
  Submitted to 2019 AMERICAN CONTROL CONFERENCE, PHILADELPHIA, USA

A single agent is represented by an holonomic mobile robot evolving within the 2D space, featuring actuated translational and rotational motion.

The translational and rotational dynamics

$$\Sigma^{I}_{tra} : \begin{cases} \dot{\xi} = \boldsymbol{v} \\ \mathbb{M}\dot{\boldsymbol{v}} + mge_z = R\boldsymbol{f}^B \end{cases} \quad (1)$$

$$\Sigma^{B}_{rot} : \begin{cases} \dot{R} = R\boldsymbol{\Omega}^{\times} \\ \mathbb{I}\dot{\boldsymbol{\Omega}} + \mathbb{I}\boldsymbol{\Omega} \times \boldsymbol{\Omega} = \boldsymbol{\Gamma} \end{cases} \quad (2)$$

The torques τ and forces f, in the body frame B, are given as follows

$$\boldsymbol{f}^B = (f_u, f_v, f_w)^T \\ \boldsymbol{\tau}^B = (\tau_p, \tau_q, \tau_r)^T \tag{3}$$

Considering that the agents are fully actuated, the dynamic equations are

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \mathbf{U}, \tag{4}$$

$$M(\boldsymbol{q}) = \begin{bmatrix} \mathbb{M} & 0 \\ 0 & \mathbb{I}\mathcal{W}_n \end{bmatrix}; C(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} \mathbb{M} & 0 \\ 0 & \dot{\mathcal{W}}_n \dot{\eta} + \mathbb{I}(\mathcal{W}_n \boldsymbol{\eta}) \times (\mathcal{W}_n \boldsymbol{\eta}) \end{bmatrix} \tag{5}$$

$$\boldsymbol{G}(\boldsymbol{q}) = \begin{bmatrix} mge_z \\ 0 \end{bmatrix}; \boldsymbol{U} = \begin{bmatrix} R\boldsymbol{f}^B \\ \boldsymbol{\tau} \end{bmatrix} \tag{6}$$

and considering the vectors

$$\boldsymbol{q} = (\boldsymbol{\xi}, \boldsymbol{\eta})^T \in \mathbb{R}^6 \tag{7}$$

We restrict the motion profile of the i^{th} agent to the horizontal plane ($e_x - e_y$ plane). This renders Eq.(4) into

$$\ddot{\boldsymbol{q}}_{(i)} = M(\boldsymbol{q})^{-1}(\boldsymbol{U}) \tag{8}$$

where

$$M(\boldsymbol{q})_{(i)} = \begin{bmatrix} \mathbb{M}_{2\times2} & 0 \\ 0 & I_y \end{bmatrix};\ \boldsymbol{U}_{(i)} = \begin{bmatrix} R\boldsymbol{f}^B \tau_\psi \end{bmatrix} \tag{9}$$

The corresponding adjacency matrix is

$$\mathcal{A} = [a_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{10}$$

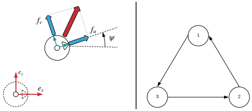


Figure: [left] Single agent freebody sketch, [right] Cyclic topology

It is possible to find an input-state linearization

$$\ddot{\boldsymbol{q}}_{(i)} = \boldsymbol{\nu} \tag{11}$$

whose control input can be split as

$$\ddot{\boldsymbol{q}}_{(i)} = \boldsymbol{\nu}_{int_{(i)}} + \boldsymbol{\nu}_{c_{(i)}} \tag{12}$$

The position/trajectory of the geometrical flock's center of mass. Hence, Eq.(13) is

$$\begin{aligned}\ddot{\boldsymbol{q}}_{(i)} &= -\Sigma_{j=1}^{m}\alpha_{ij}(\boldsymbol{q}_{(i)} - \boldsymbol{q}_{(j)}) \\ &\quad -\Sigma_{j=1}^{m}\beta_{ij}(\dot{\boldsymbol{q}}_{(i)} - \dot{\boldsymbol{q}}_{(j)}) + \ddot{\boldsymbol{q}}_{(j)} + \boldsymbol{\nu}_{c_{(i)}}\end{aligned} \tag{13}$$

The model for the multi-agent system considering the time delay

$$\begin{aligned}\ddot{\boldsymbol{q}}_{(i)}(t) &= -\Sigma_{j=1}^{m}\alpha_{ij}(\boldsymbol{q}_{(i)}(t-\tau_{1_{ij}}) - \boldsymbol{q}_{(j)}(t-\tau_{1_{ij}})) \\ &\quad -\Sigma_{j=1}^{m}\beta_{ij}(\dot{\boldsymbol{q}}_{(i)}(t-\tau_{1_{ij}}) - \dot{\boldsymbol{q}}_{(j)}(t-\tau_{1_{ij}})) \\ &\quad +\ddot{\boldsymbol{q}}_{(j)}(t) + \boldsymbol{\nu}_{c_{(i)}}\end{aligned} \tag{14}$$

After denoting the consensus protocol in terms of error and taking the Laplace transform, one obtains

$$s^2E(s) + \Sigma\alpha e^{-\tau_{1ij}s}E(s) + \Sigma\beta s e^{-\tau_{1ij}s}E(s) = 0 \tag{15}$$

Then, the corresponding quasipolynomial function is given by

$$\triangle(s,\tau) = s^2 + \alpha e^{-s\tau} + \beta s e^{-s\tau} \tag{16}$$

where $\beta \neq 0$.

For any $\tau \in \mathbb{R}_+$ there exist a stabilizing pair $(\alpha, \beta) \in \mathbb{R}^2_+$.

$$\alpha(\sigma) = (2 - \tau\sigma)\sigma e^{-\sigma\tau}, \tag{17}$$
$$\beta(\sigma) = (1 - \tau\sigma)\sigma^2 e^{-\sigma\tau}. \tag{18}$$

Then, Δ is asymptotically stable whenever $0 < \sigma < \frac{1}{\tau}$. If in addition $\sigma = \frac{2-\sqrt{2}}{\tau}$, then such a solution is dominant.

Control Strategy
Multi-agent Formation Control

Each agent consists of an attitude and position controllers. Now, considering the state $q = (\xi, \dot{\xi}, \eta_x, \dot{\eta}_x, \eta_y, \dot{\eta}_y)$ for every agent, the average attitudes and linear positions are

$$\xi_C = \frac{1}{N}\sum_{i=1}^{N}\xi_i \quad \dot{\xi}_C = \frac{1}{N}\sum_{i=1}^{N}\dot{\xi}_i \tag{19}$$

$$\eta_{CM_x} = \frac{1}{N}\sum_{i=1}^{N}\eta_{i_x} \quad \dot{\eta}_{CM_x} = \frac{1}{N}\sum_{i=1}^{N}\dot{\eta}_{i_x} \tag{20}$$

$$\eta_{CM_y} = \frac{1}{N}\sum_{i=1}^{N}\eta_{i_y} \quad \dot{\eta}_{CM_y} = \frac{1}{N}\sum_{i=1}^{N}\dot{\eta}_{i_y} \tag{21}$$

Assuming that the agent 1 is the leader, a consensus algorithm to reach an agreement for orientation and position can be given by

$$u_{1_\xi} = -\lambda_1 sat(\xi_C^d - \xi_C) - \lambda_2 sat(\dot{\xi}_C^d - \dot{\xi}_C) \tag{22}$$

$$u_{1x} = -\lambda_3 sat(\eta_{CM_x}^d - \eta_{CM_x}) - \lambda_4 sat(\dot{\eta}_{CM_x}^d - \dot{\eta}_{CM_x}) \tag{23}$$

$$u_{1y} = -\lambda_5 sat(\eta_{CM_y}^d - \eta_{CM_y}) - \lambda_6 sat(\dot{\eta}_{CM_y}^d - \dot{\eta}_{CM_y}) \tag{24}$$

Let $\triangle$ be a set of relative, desired inter-agent distances, that is

$$\triangle = [\delta_{ij} \in \mathbb{R} | \delta_{ij} > 0; \quad i, j = 1, ..., N, i \neq 0] \tag{25}$$

It is possible to extend the consensus algorithms to formation control if the formation is represented by vectors of relative attitudes or linear positions of neighboring agents.

$$\ddot{\boldsymbol{q}}_{(i)} = -\Sigma a_{ij} K(\boldsymbol{q}_{(i)} - \boldsymbol{q}_{(j)} - \delta_{ij}) - \Sigma a_{ij} K(\dot{\boldsymbol{q}}_{(i)} - \dot{\boldsymbol{q}}_{(j)}) + \ddot{\boldsymbol{q}}_{(j)} + \boldsymbol{\nu}_{c_{(i)}} \tag{26}$$

with $K \in \mathbb{R} > 0$

The simulation consists on three stages.

- The robot multi-agent system is initialized at the states given by the Table 1. Then, it is sent to $p_o = [1\ 1]^T m$. Orientation consensus is performed to $\xi_i^d = (-90° \ -210° 30°)$. The contact points are located at $\eta_1^d = [0.95\ 0.99]^T m$, $\eta_2^d = [1\ 1.085]^T m$ and $\eta_3^d = [1.05\ 0.99]^T m$.
- A trajectory tracking consensus is performed between the time 15s and 60s and it is given by $\eta_{CM_x}^d = 0.5\sin(0.07t + 1.5)$ and $\eta_{CM_y}^d = 0.25\sin(0.1t + 1.27)$.
- When the object center of mass reaches the desired place, given by $p_{od} = [2\ 1.5]^T m$, the multi-agent system drops the object.

Table: Initial conditions

Robot	Orientation (ξ_i)	Position η_{i_x}	Position η_{i_y}
1	15°	0.85m	−0.6m
2	5°	0.1m	0.25m
3	-10°	−0.6m	1.05m

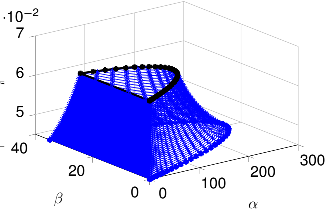


Figure: $3D-$ Region in the $\alpha - \beta$ parameter space, for $\tau \in [0.045, 0.065]$.

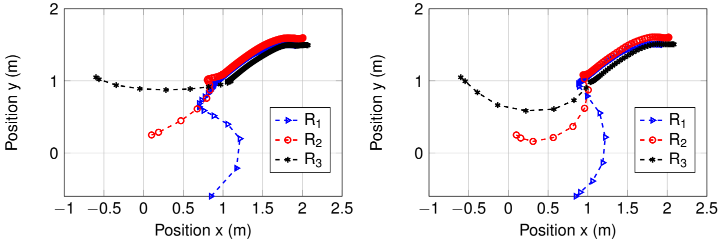
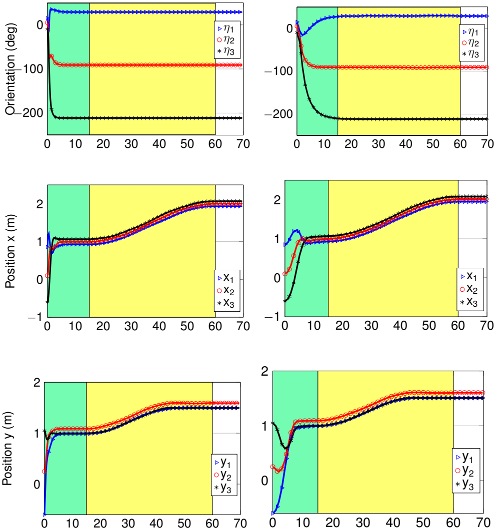


Figure: 2*d* positions of the three agents during the simulation.(Top) without communication delay, (bottom) with communication delay

Simulation Results

Simulation Results



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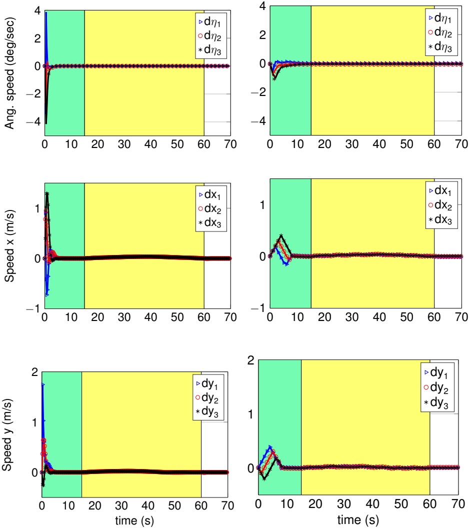

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Conclusions and Perspectives

Conclusions

- A control methodology is described to stabilize the collective motion (consensus) regarding the transport operations.
- Such a control strategy consists on a time-delay tolerant control law, which guarantees the stabilization of an individual agent.
- Explicit stability regions for a range of delays concerning the proposed quasipolynomial is presented and explained.
- Simulation results show the effectiveness of the proposed multi-agent control methodology for an object transport scenario.
- The simulation allows also to perform a comparison between a time-delay multi-agent system and simple multi-agent system.

Perspectives

- Explicit presentation and explanation of the object contact points and coordinated object avoidance will be discussed.

Thank You