Forced Bipartite Consensus for Multi-agent Systems

J.A. Guerrero, D. Olivares, G. Romero

GT UAV

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Compiègne

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Introduction

- Multi-agent consensus problem is present in numerous applications, both in natural and in man-made systems.

Flocking of Birds

Schools of Fish

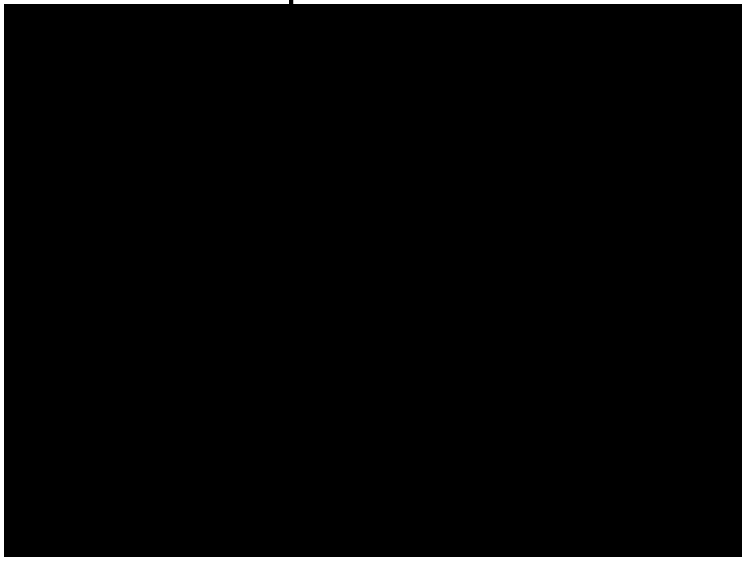
Crowd Dynamics

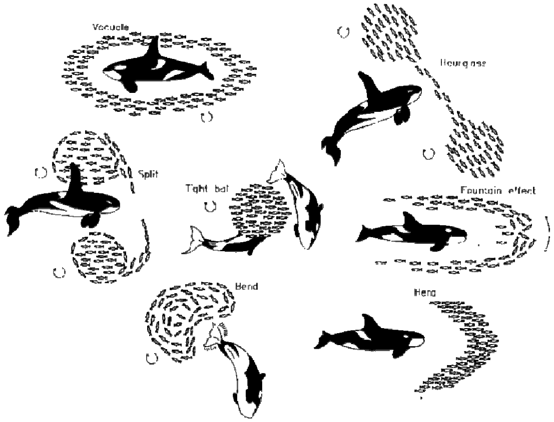
Formation Flying and UAV Networks

Convoy of Trucks

Introduction

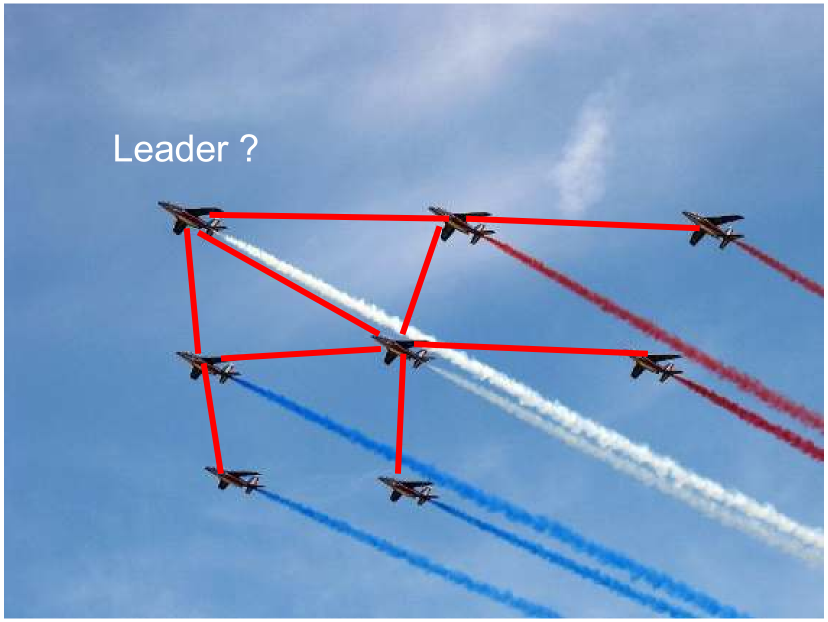
- Antagonist and anti-predator behavior can be modeled using multi-agent consensus problems.



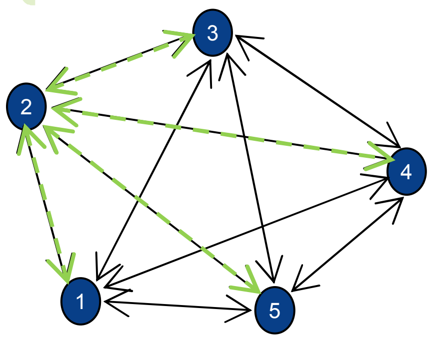


Fundamental Questions

1) How to model antagonistic behavior on multi-agent systems?
2) Are the controllability / observability properties of a leader-based multi-agent system affected by antagonistic interactions?
3) Design of a forced bipartite consensus control?

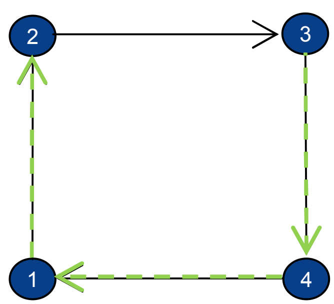


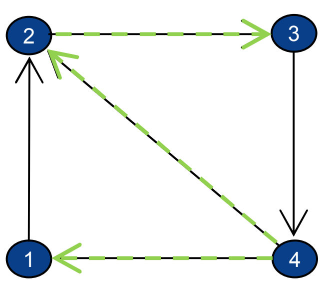
Communication Graphs



Balanced, Undirected

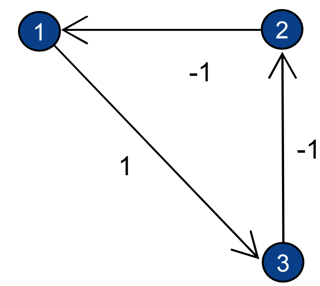
Balanced, Directed

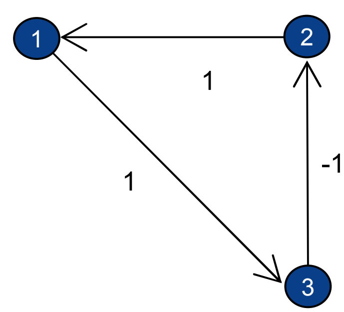




Not Balanced, Directed

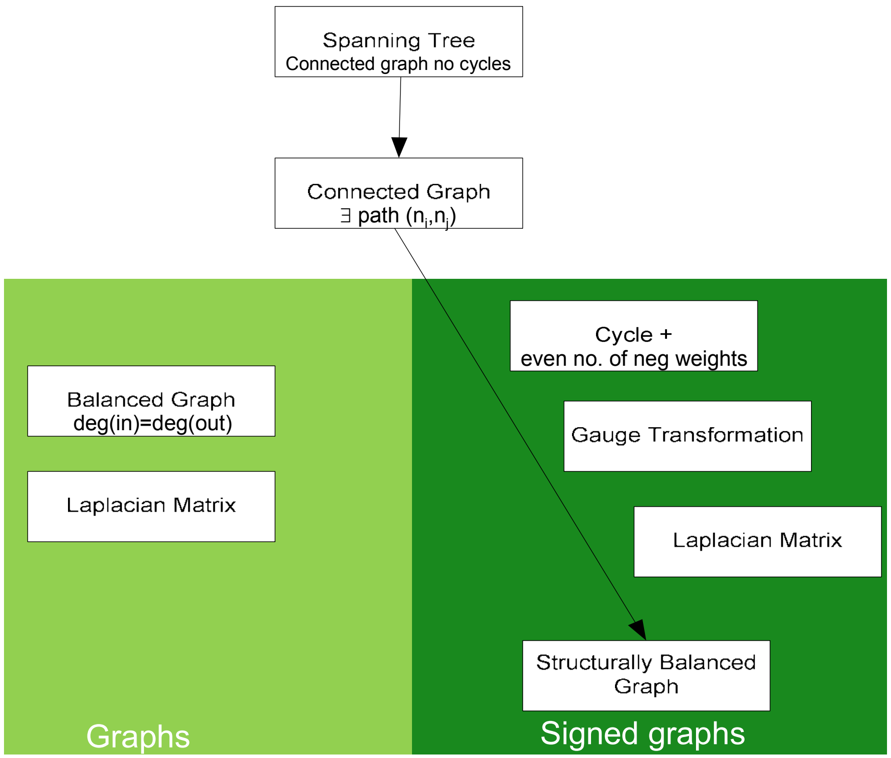
Directed, Cycle+,
Structurally Balanced





Directed, Cycle-,
Structurally Unbalanced

Communication Graphs



Definition 1: Let vector $\sigma = [\ \sigma_1 \ \ \ldots \ \ \sigma_n\ ]$ with $\sigma_i \in \{\pm 1\}$ be a partial orthant order in $\mathbb{R}_n$. A gauge transformation is a change of orthant order in $\mathbb{R}_n$ performed by a matrix $D = diag(\sigma)$ and the family of gauge transformations in $\mathbb{R}_n$ is defined as $\mathcal{D} = \{D \in \mathcal{D}\}$.

- $D^{-1} = D = D^T$ and $|\ \det D| = 1$.
- The eigenvalues of a matrix $\mathcal{L}$ under a gauge transformation $\mathcal{L} = D\mathcal{L}_D D$ are all preserved, i.e. sp($\mathcal{L}$)=sp($\mathcal{L}_D$).

Laplacian Matrix: $\mathcal{L} = \mathcal{C}$-$\mathcal{A}$ where $\mathcal{C}$ is the degree matrix and $\mathcal{A}$ is the adjacency matrix.

1) $\mathcal{L}$ has a single eigenvalue at 0, $\lambda_1(\mathcal{L}) = 0$ with right eigenvector $w_1^T = [\ 1 \quad 1 \quad \cdots \quad 1\ ]$,i.e. $\mathcal{L}w_1 = 0$.

2) The remaining eigenvalues have all positive real part, i.e. $Re[\lambda_i(\mathcal{L})] > 0$ and $\mathcal{L}w_i = \lambda_i w_i$ for *i* = 2,…,*n;* and $w_i \in R^n$

3) w_1 defined above is also the left eigenvalue of $\mathcal{L}$ corresponding to the eigenvalue 0*;* i.e. $w_1^T\mathcal{L} = 0$

Multi-agent Consensus and Bipartite Consensus

Consider a single integrator multi-agent system:

$$\dot{\xi}_i = u_i \qquad (1)$$

• Average consensus control algorithm is defined as:

$$u_i \triangleq k_p \sum_{j \in \mathcal{N}_i} (\xi_j - \xi_i) \qquad (2)$$

Definition 2: ([Murray, Olfati]) Let A be constant. Algorithm (2) achieves average consensus asymptotically if and only if directed graph G is strongly connected and balanced or undirected graph G is connected.

• Average bipartite consensus control algorithm is defined as:

$$\dot{x}_i = -\sum_{j \in \mathrm{adj}(i)} |a_{ij}| \left(x_i - \mathrm{sgn}(a_{ij}) x_j\right)$$

Definition 3: ([Altafini]) Let G the information graph of a MAS with antagonistic interactions. The multi-agent system (1) with consensus algorithm (3) admits a bipartite consensus solution if

$$\lim_{t \to \infty} |\xi_i| = \alpha > 0 \ \forall i = 1 \ \dots \ n$$

Leader based Multi-agent System

- Consider the forced consensus algorithm:

$$u_i \triangleq k_p \sum_{j \in \mathcal{N}_i} (\xi_j - \xi_i) + b_i u_l \qquad (3)$$

- The MAS can be rewritten as

$$\dot{\xi} = -k_p \mathcal{L} \xi + b u_l \qquad (4)$$

$$y = c^T \xi \qquad (5)$$

- where u_l is the input given only to the leader and input and ouput vectors are defined as follows:

$$b = c^T = [\ 0 \ \dots \ 1 \ \dots \ 0\ ]$$

Controllability & Observability w/ Antagonistic Interactions

Proposition 1: The center of mass of a multi-agent system with antagonistic interactions corresponds to controllable and observable modes if and only if the corresponding connected signed graph G is structurally balanced.

Proof: If G is structurally balanced there $\exists D \in \mathcal{D}$ such that $D\mathcal{A}D \geq 0.$ That implies that $\lambda_1(\mathcal{L}) = 0$ is an eigenvalue of L with right eigenvector $w_1 = D1$, i.e. $\mathcal{L}w_1 = 0.$ Due to the fact that $c^T w_1 \neq 0$, the corresponding mode (λ_1, w_1) is observable. Since L is symmetric then L and L^T have the same eigenvalues, w_1 is also the left eigenvector of L corresponding to the eigenvalue $\lambda_1(\mathcal{L}) = 0$, i.e. $w_1^T \mathcal{L} = 0.$ Let v_i be the left eigenvector of L, then $v_i^T \mathcal{L} = \lambda_i v_i^T.$ Premultiplying (4) by v_i^T we obtain

$$\frac{d}{dt}(v_i^T x) = -\lambda_i(v_i^T \xi) + v_i^T b u_l \qquad (6)$$

Since $v_1 = w_1,$ then $v_1 b \neq 0.$ Thus the mode (λ_1, v_1) corresponds to a controllable mode. If there is a $c^T w_1 = 0$ or $v_i^T b = 0$ means that the corresponding modes are not observable or not controllable, respectively.

Forced Bipartite Consensus

Example: Consider the gauge transformed MAS:

$$\dot{\eta}_i = \bar{u}_i$$

With control

$$\bar{u}_i \triangleq \sigma_i k_p \sum_{j \in \mathcal{N}_i} (\eta_j - \eta_i) + b_i u_i$$

Premultiply by its eigenvectors

$$\dot{\eta}_1 + \dot{\eta}_2 + \dot{\eta}_3 = u_l$$

$$\dot{\eta}_1 - 2\dot{\eta}_2 + \dot{\eta}_3 = -3(\eta_1 - 2\eta_2 + \eta_3) + u_l$$

$$\dot{\eta}_1 - \dot{\eta}_3 = -(\eta_1 - \eta_3) + u_l$$

Foced Bipartite Consensus

The center of mass is defined as:

$$\eta_{CM} \triangleq \frac{1}{3}\sum_{i=1}^{3}\eta_i = \sum_{i=1}^{3}\sigma_i\eta_i = \frac{1}{3}D\xi$$

We define:

$$u_1 = Nk_{cm}\rho(\eta_{CM}^d - \eta_{CM})$$

If the desired velocity value η_{CM}^d is constant, then $\eta_{CM} \longrightarrow \eta_{CM}^d$ as $t \longrightarrow \infty$, which implies that $u_1 \longrightarrow 0$ and $(\eta_1 - \eta_3), (\eta_1 - \eta_2) \longrightarrow 0$. Finally, since $\xi = D\eta$, we can conclude that $(|\eta_i| - |\eta_j|) \longrightarrow 0$.

Forced Bipartite Consensus

Definition 4: Let ξ^d be a desired reference for a MAS. Forced bipartite consensus of MAS with bipartite consensus algorithm

$$\bar{u}_i \triangleq \sigma_i k_p \sum_{j \in \mathcal{N}_i} (\eta_j - \eta_i) + b_i u_i$$

is said to be achieved if for any initial states

$$\lim_{t \to \infty} |\xi_i(t) - \xi^d(t)| = 0 \ \forall i = 1, ..., N.$$

Lemma 2: Consider a multi-agent system of the form (4) with antagonistic interactions, a spanning tree from the leader and coordinating control law (11). If η_{CM}^d is constant, then $\eta_{CM} \longrightarrow \eta_{CM}^d$ as $t \longrightarrow \infty$, which implies that $u_l \longrightarrow 0$ and $(\eta_i - \eta_j) \longrightarrow 0.$ Moreover, since there exists a spanning tree from the leader, $\lim_{t\to\infty} \eta_i = \eta_{CM}^d$ due to the fact that η_{CM}^d is constant. Using the inverse gauge transformation, we conclude that $\lim_{t\to\infty}(|\xi_i| - |\eta_{CM}^d| = 0$

Quadrotor Dynamic Model

- The simplified dynamic model of a quadrotor is:

$$\ddot{x}_i = -T_i \sin(\theta_i) \qquad (17)$$

$$\ddot{y}_i = T_i \cos(\theta_i)\sin(\phi_i) \qquad (18)$$

$$\ddot{z}_i = T_i \cos(\theta_i)\cos(\phi_i) - 1 \qquad (19)$$

$$\ddot{\theta}_i = \tilde{\tau}_{\theta,i} \qquad (20)$$

$$\phi_i = \tilde{\tau}_{\phi,i} \qquad (21)$$

$$\psi_i = \tilde{\tau}_{\psi,i} \qquad (22)$$

- The nonlinear nested saturations control:

$$T_i = \frac{-k_{1,i}\dot{z}_i - k_{2,i}(z_i - z_i^{\,d}) + 1}{\cos(\phi_i)\cos(\theta_i)} \qquad (23)$$

$$\tilde{\tau}_{\psi,i} = -k_{3,i}\dot{\psi}_i - k_{4,i}(\psi_i - \psi_i^{\,d}) \qquad (24)$$

$$\tilde{\tau}_{\theta,i} = -\sigma_4(\dot{\theta}_i) - \sigma_3(\dot{\theta}_i + \theta_i) - \sigma_2(\dot{\theta}_i + 2\theta_i - \dot{x}_i) - \sigma_1(\dot{\theta}_i + 3\theta_i - 3\dot{x}_i - x_i) \qquad (25)$$

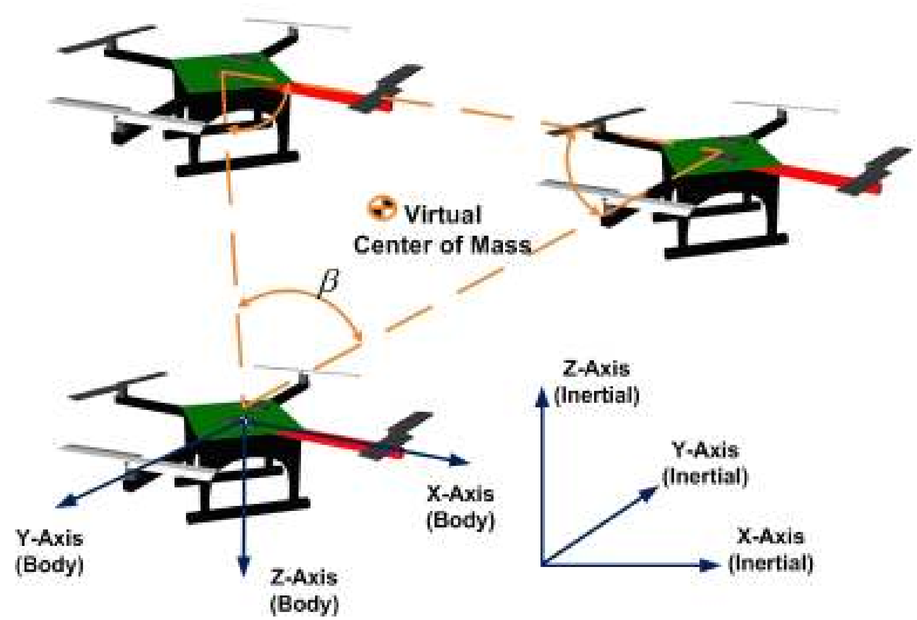
$$\tilde{\tau}_{\phi,i} = -\sigma_4(\dot{\phi}_i) - \sigma_3(\dot{\phi}_i + \phi_i) - \sigma_2(\dot{\phi}_i + 2\phi_i + \dot{y}_i) - \sigma_1(\dot{\phi}_i + 3\phi_i + 3\dot{y}_i + y_i) \qquad (26)$$

Quadrotor Platoon with Antigonistic Interactions

- Quadrotor bipartite consensus control:

$$T_i = \frac{-k_{1,i}\dot{z}_i - k_{2,i}(\sum_{j\in N_i}(z_i - z_j) - z_i^{\,d}) + 1}{\cos(\phi_i)\cos(\theta_i)} \quad (28)$$

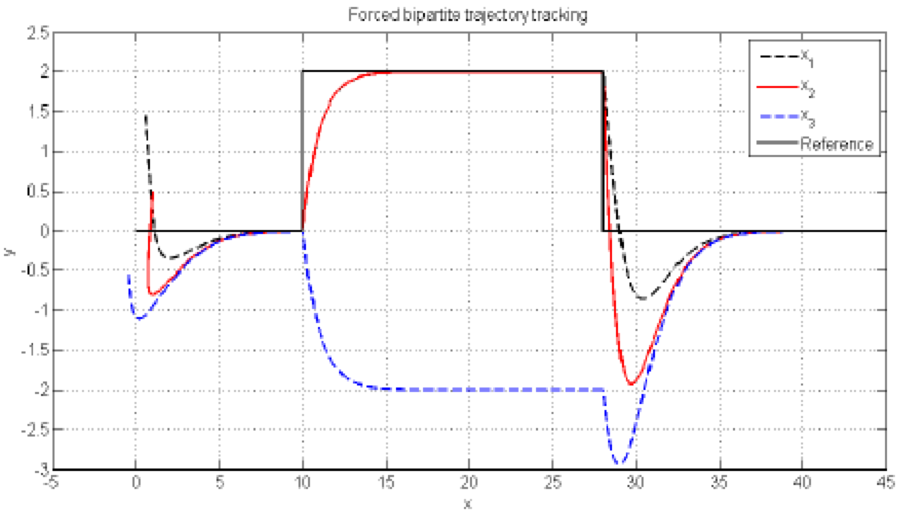
$$\tilde{\tau}_{\psi,i} = -k_{3,i}\dot{\psi}_i - k_{4,i}(\sum_{j\in N_i}(\psi_i - \psi_j) - \psi_i^{\,d}) \quad (29)$$



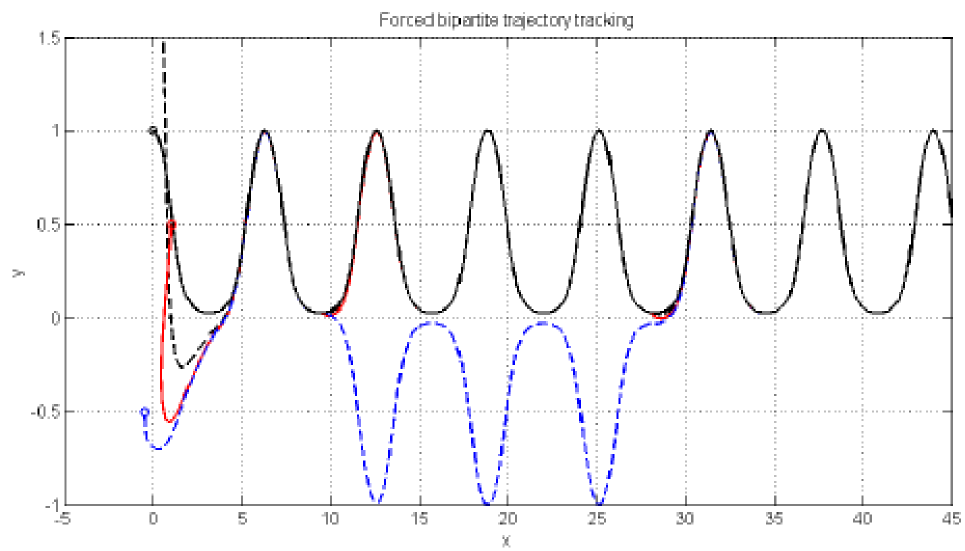
$$\tilde{\tau}_{\theta i} = -\sigma_4(\dot{\theta}) - \sigma_3(\dot{\theta} + \theta) - \sigma_2(\dot{\theta} + 2\theta - \dot{x})$$
$$-\sigma_1\left(\dot{\theta} + 3\theta - 3\dot{x} - \sigma_i^{\,x} k_p \sum_{j\in N_i}(x_j - x_i) + u_i^{\,x}\right) \quad (30)$$

$$\tilde{\tau}_{\phi i} = -\sigma_4(\dot{\phi}) - \sigma_3(\dot{\phi} + \phi) - \sigma_2(\dot{\phi} + 2\phi + \dot{y})$$
$$-\sigma_1\left(\dot{\phi} + 3\phi + 3\dot{y} - \sigma_i^{\,y} k_p \sum_{j\in N_i}(y_j - y_i) + u_i^{\,y}\right) \quad (31)$$

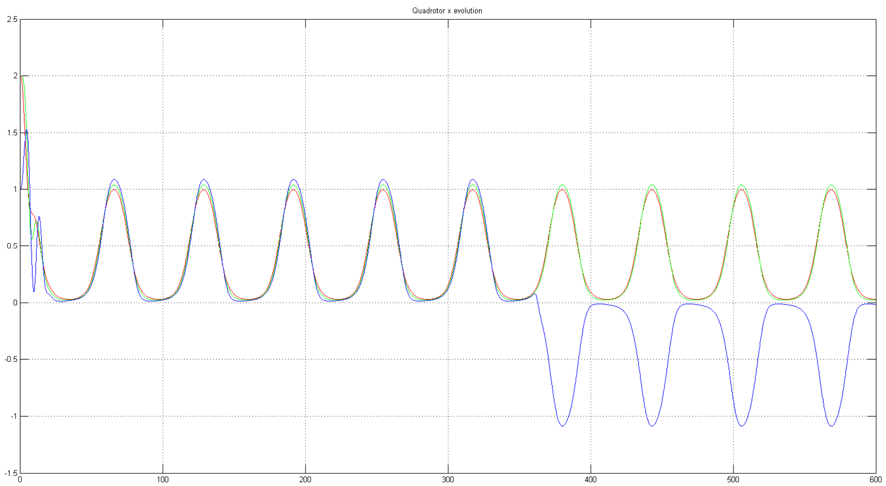
Simulation Results (Single Integrator)



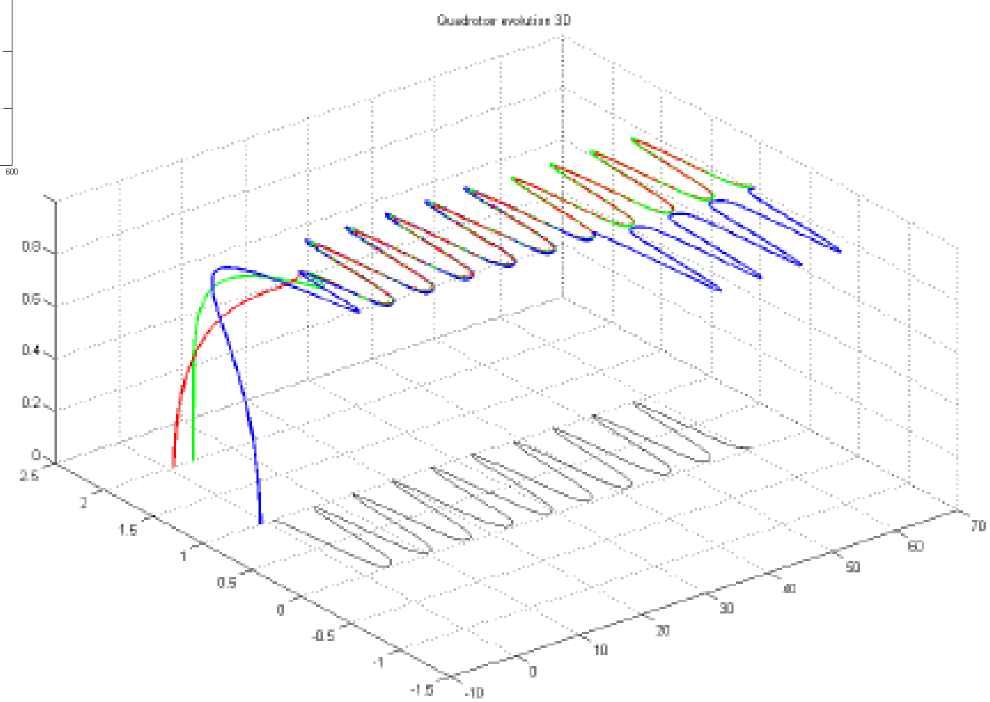
Chain Topology



Simulation Results (Quadrotor Platoon)



Chain Topology



Conclusions and Future Work

- A switching formation control based on the forced bipartite consensus over signed graphs or networks with antagonistic interactions has been presented.
- It has been shown that the controllability and observability of the center of mass of a multi-agent system is not affected by antagonistic interactions.
- Tracking of a constant reference is achieved using the center of mass of the MAS.
- Tracking of the desired reference for the center of mass is achieved using a full state feedback control on the leader.
- A potential application of forced bipartite consensus has been tested on a quadrotor platoon on simulation.
- Future work include bipartite consensus on high-order multi-agent systems, bipartite consensus and experimental validation on multi-robot systems.

Questions ?