Analysis, Configuration Design and Control of an Aerial Cable-Towed System

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GT-UAV Presentation

12 octobre 2018

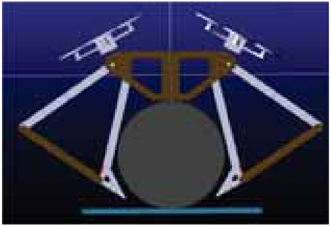
Presentation Outline

The Authors

- **Julian Erskine**
  - Master Student, ECN (2016-2018)
  - PhD Student, ECN (2018-2021)
  - Multi-UAV Systems, Parallel Robotics
  - Decentralized Control, Swarm Rigidity, Formation Singularities
- **Abdelhamid Chriette**
  - Teacher, ECN
  - Researcher, LS2N
  - Controls, Robotics
- **Stéphane Caro**
  - CNRS Researcher, LS2N
  - Institute de Recherche Technique Jules Verne
  - Cable-Driven Parallel Robots, Parallel Mechanisms

Laboratoire des Sciences du Numérique de Nantes

- **Robotics**, **Control**, Signal Processing, and Data Science
  - Serial and Parallel Robots
  - Advanced Manufacturing Robots
  - Autonomous Vehicles
  - Interactions with Environment
- Unmanned Aerial Vehicles
  - Parallel Manipulators
  - Novel Applications
  - Swarms



Zhongmou Li

Damien Six

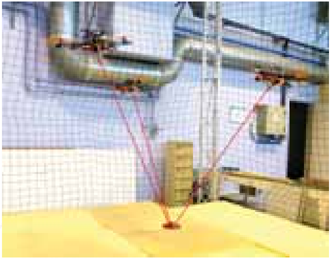
UAV Applications

- Commercial UAVs : 110k in 2017, 450k in 2022 (U.S.A) [1]
  - Aerial Photography (48%)
  - Industrial Inspection (28%)
  - Agriculture (17%)
  - Other (7%)
- Perception-based tasks
- Developing uses : Construction and logistics
- Connecting quadrotors with cables
  - Long and lightweight
  - Decouple translation and rotation
  - Modular and easily adaptable

1. https://www.faa.gov/data_research/aviation/aerospace_forecasts/media/Unmanned_Aircraft_Systems.pdf, 30/07/2018

Objectives

- Study how ACTS interact with the environment.
  - Configuration Planning
  - ACTS Design
  - Wrench Limits
- Generalize wrench capabilities as function of :
  - Quadrotor type
  - Cable Connectivity
  - Payload
- Scope : Quasi-static quadrotor motion
- Similar to cable-driven parallel robots
  - Cable tension constraints
  - Kinematics
  - Control



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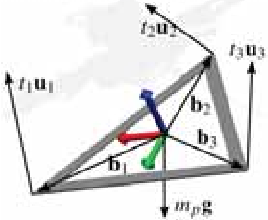
Dynamics of Platform and Quadrotor

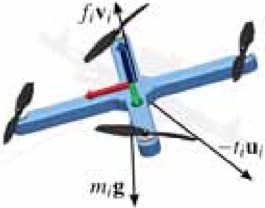
- n quadrotors, m cables, d DOF platform
  - $d \leq m$
  - $n \leq m \leq 2n$
- Massless, positive tension cables



- $\mathbf{Wt} + m_p\mathbf{g} + \mathbf{w}_e = m_p\ddot{\mathbf{x}}_p$

$$\mathbf{W} = \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_m \\ \mathbf{b}_1 \times \mathbf{u}_1 & \cdots & \mathbf{b}_m \times \mathbf{u}_m \end{bmatrix}$$
$$\mathbf{t} = [t_1 \cdots t_m]^T$$

- Cable passes through COM
- Actuation : $[f_z, m_x, m_y, m_z]$



- $f_i\mathbf{v}_i + m_i\mathbf{g} - t_i\mathbf{u}_i = m_i\ddot{\mathbf{x}}_i$

Thrust, Tension, and Wrench Spaces

- Thrust space of n quadrotors

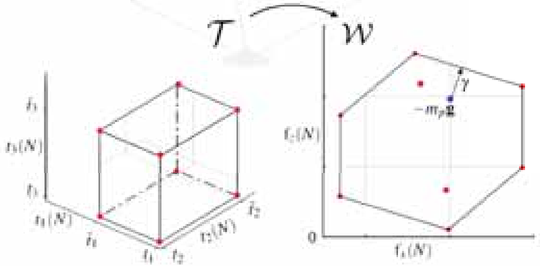
$$\mathcal{H} = \left\{ \mathbf{f} \in \mathbb{R}^n : \underline{\mathbf{f}} \leq \mathbf{f} \leq \bar{\mathbf{f}} \right\}, \qquad \mathbf{f} = [f_1, \cdots, f_n]^T \tag{1}$$

- Tension space of m cables

$$\mathcal{T} = \left\{ \mathbf{t} \in \mathbb{R}^m : 0 < \underline{\mathbf{t}} \leq \mathbf{t} \leq \bar{\mathbf{t}} \right\}, \quad \mathbf{t} = [t_1, \cdots, t_m]^T \tag{2}$$

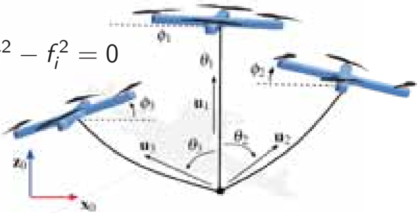
- Wrench space in d DOF

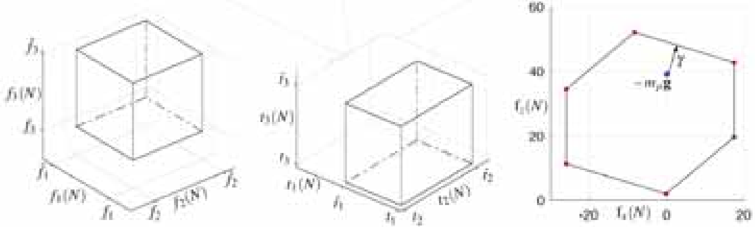
$$\mathcal{W} = \left\{ \mathbf{w} \in \mathbb{R}^d | \mathbf{w} = \sum_{j=1}^{m} \alpha_j \Delta t_j \mathbf{w}_j + \mathbf{W}\underline{\mathbf{t}}, \quad 0 \leq \alpha_j \leq 1 \right\} \tag{3}$$



Uncoupled Tension ACTS

- Determine $\bar{t}_i$
  - CDPR : Limited by winch torque or cable strength
  - ACTS : Limited by quadrotor thrust
- Thrust constraint : $t_i^2 - 2m_i\mathbf{g}^T t_i\mathbf{u}_i + m_i^2 g^2 - f_i^2 = 0$
- $\bar{t}_i = m_i\mathbf{g}^T\mathbf{u}_i + \sqrt{\bar{f}_i^2 + m_i^2 g^2\,(u_{iz}^2 - 1)}$

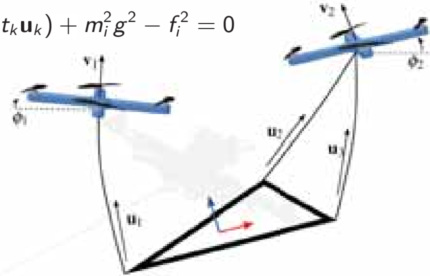




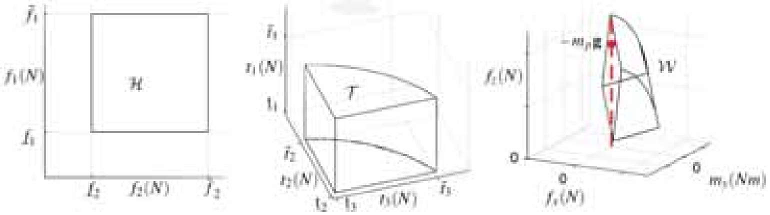
A) THRUST SPACE B) TENSION SPACE C) WRENCH SPACE

Coupled Tension ACTS

- Two cables (j, k) to a single quadrotor (i)
  - Thrust constraint :
    $t_j^2 + t_k^2 + 2t_j t_k \left(\mathbf{u}_j^T \mathbf{u}_k\right) - 2m_i \mathbf{g}^T (t_j \mathbf{u}_j + t_k \mathbf{u}_k) + m_i^2 g^2 - f_i^2 = 0$
  - More DOF with fewer quadrotors
  - Intuitively wider range of moments

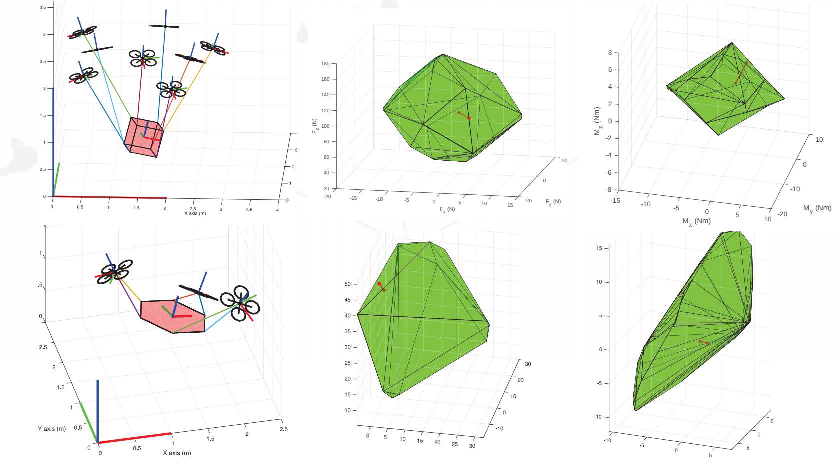


$$\mathcal{T} = \mathbf{t} \in \mathbb{R}^3 \begin{cases} \underline{t}_1 \leq t_1 \leq \overline{t}_1 \\ \underline{t}_2 \leq t_2 \leq h_2(t_3, \overline{f}_2) \\ \underline{t}_3 \leq t_3 \leq h_3(t_2, \overline{f}_2) \end{cases}$$



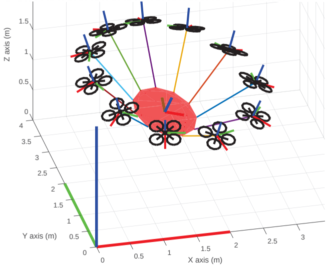
Comparing Designs

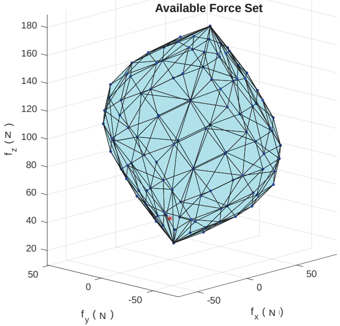
- Application-specific objectives
  - Vertical Lift
  - Isotropic wrench set
  - Required platform pose

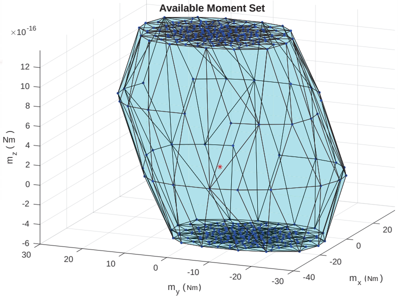


Singular Configurations

- Rank deficient $\mathbf{W}$
- All vectors $\mathbf{u}_i$ interest at a point.
  - Well know singularity
  - Unconstrained Moment about $\mathbf{z}_0$

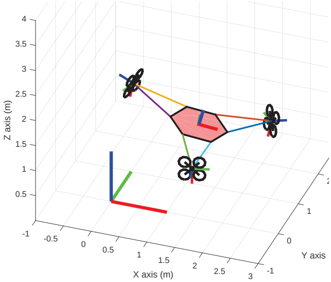


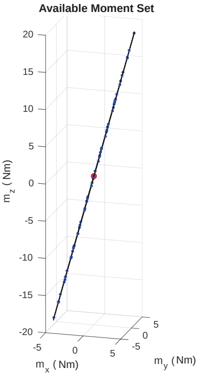


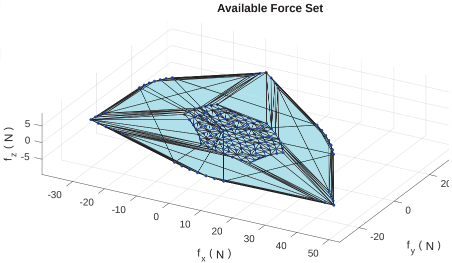


Singular Configurations

- Rank deficient $\mathbf{W}$
- All vectors $\mathbf{u}_i$ are co-planar.
  - Forces along plane of cables
  - Moment about $\mathbf{z}_p$

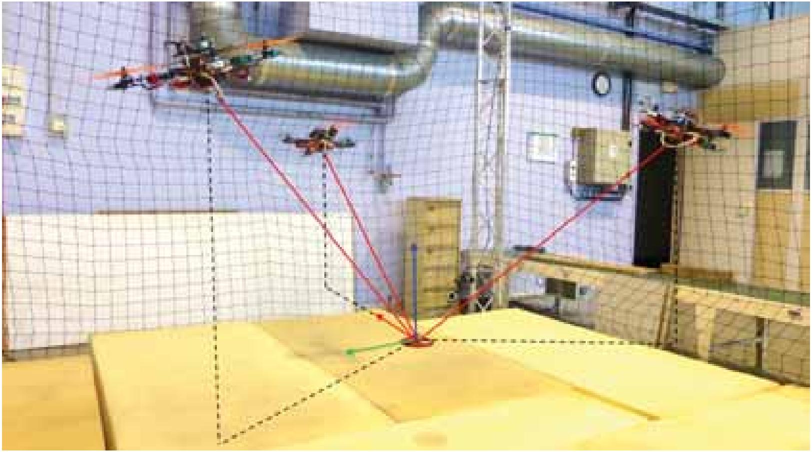






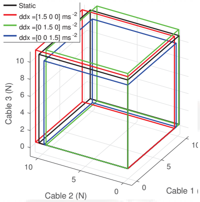
Case Study

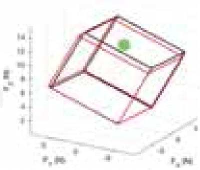
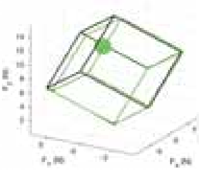
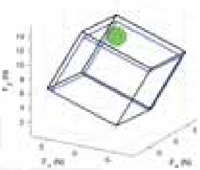
Prototype used to validate models :



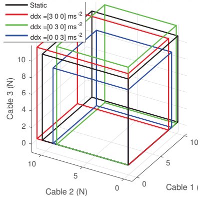
Effect of Dynamics on Wrench Space

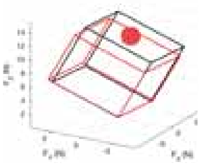
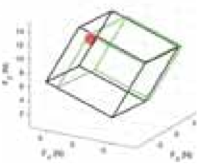
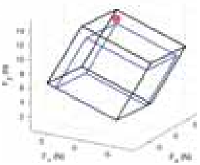
Tension Space $\ddot{\mathbf{x}}_p = [1.5\ 0\ 0]^T$, $\ddot{\mathbf{x}}_p = [0\ 1.5\ 0]^T$, $\ddot{\mathbf{x}}_p = [0\ 0\ 1.5]^T$



Tension Space $\ddot{\mathbf{x}}_p = [4\ 0\ 0]^T$, $\ddot{\mathbf{x}}_p = [0\ 4\ 0]^T$, $\ddot{\mathbf{x}}_p = [0\ 0\ 4]^T$

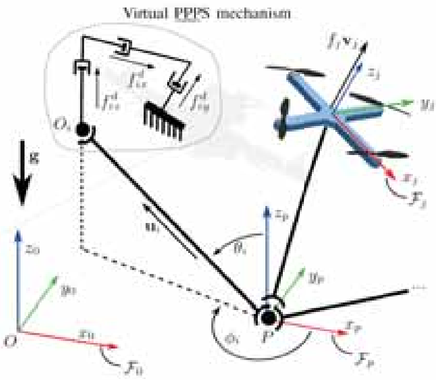


Geometric Modelling

- Virtual PPPSS Mechanism
  - Quadrotor translates in 3D
  - Cable can't support moments
  - Cable is constant length
- Mapping of State **X**
  - Measure $\mathbf{x}_p = \vec{OP}$, $\mathbf{x}_i = \vec{OO_i}$
  - Control $\mathbf{X} = [\mathbf{x}_p, \mathcal{C}]$
  - $\mathcal{C} = \begin{bmatrix} \phi_1 & \theta_1 & \phi_2 & \theta_2 & \phi_3 & \theta_3 \end{bmatrix}$
- Required mappings :
  - $\theta_i = cos^{-1}\left(\frac{x_{i,z} - x_{p,z}}{l_i}\right)$
  - $\phi_i = atan2\left(x_{i,y} - x_{p,y}, x_{i,x} - x_{p,x}\right)$
  - $\hat{\mathbf{x}}_p = \text{DGM}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$

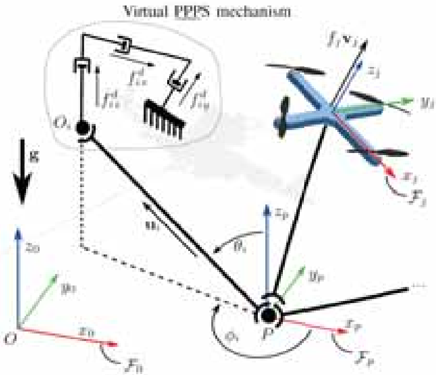
Loop closure : $\mathbf{x}_i = \mathbf{x}_p + l_i \mathbf{u}_i$



Kinematic Modelling

- DKM1
  - $\dot{\mathbf{x}}_i = \dot{\mathbf{x}}_p + l_i \dot{\mathbf{u}}_i$
  - $\dot{\mathbf{X}} = \mathbf{J} \begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\mathbf{x}}_3 \end{bmatrix}$
- DKM2
  - $\ddot{\mathbf{x}}_i = \ddot{\mathbf{x}}_p + l_i \ddot{\mathbf{u}}_i$
  - $\ddot{\mathbf{X}} = \mathbf{J} \begin{bmatrix} \ddot{\mathbf{x}}_1 \\ \ddot{\mathbf{x}}_2 \\ \ddot{\mathbf{x}}_3 \end{bmatrix} + \mathbf{b}$

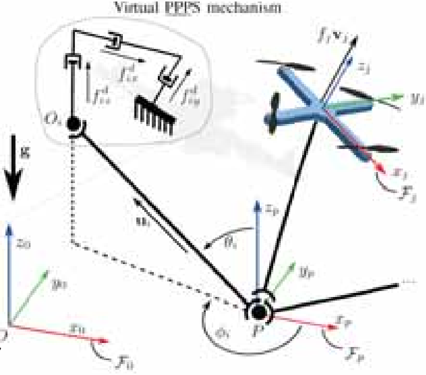
J and **b** are defined in the appendix



Dynamic Modelling

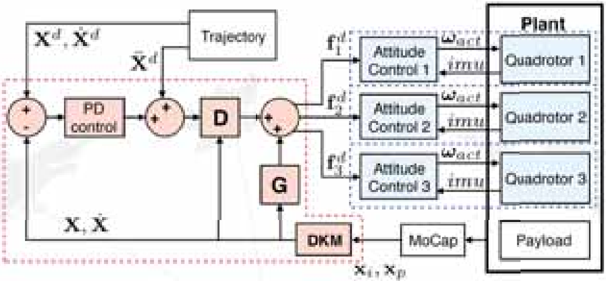
- $\mathbf{f} = [\mathbf{f}_1^T, \mathbf{f}_2^T, \mathbf{f}_3^T]^T = \text{IDM}(\mathbf{X}, \dot{\mathbf{X}}, \ddot{\mathbf{X}})$
  - Quadrotor dynamics :
    $f_i \mathbf{v}_i + m_i \mathbf{g} - t_i \mathbf{u}_i = m_i \ddot{\mathbf{x}}_i$
  - Payload dynamics
    $$m_p \mathbf{g} + \sum_{j=1}^{3} (t_j \mathbf{u}_j) + \mathbf{w}_e = m_p \ddot{\mathbf{x}}_p$$
  - Cable link
    $\mathbf{t} = -\mathbf{W}^{-1}(m_p \mathbf{g} + \mathbf{w}_e)$

$$\mathbf{f} = \underbrace{\left(\mathbf{M}_Q \mathbf{J}^{-1} + \mathbf{T}_{\ddot{x}_p}\right)}_{\mathbf{D}} \ddot{\mathbf{X}} \underbrace{- \mathbf{M}_Q \left(\begin{bmatrix} \mathbf{g} \\ \mathbf{g} \\ \mathbf{g} \end{bmatrix} + \mathbf{J}^{-1} \mathbf{b} \right) - \mathbf{T}_g}_{\mathbf{G}}$$



$\mathbf{M}_Q$, $\mathbf{T}_{\ddot{x}_p}$, $\mathbf{T}_g$ defined in appendix

Backstepping Controller

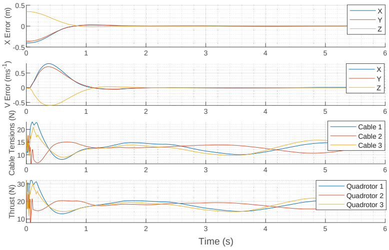


- Outer Loop
  - Choose thrust vectors $\mathbf{X} \rightarrow \mathbf{X}^d$
  - $\mathbf{f}^d = \mathbf{D}\left(\ddot{\mathbf{X}}_d + k_D\dot{\mathbf{e}} + k_P\mathbf{e}\right) + \mathbf{G}$
- Inner Loop
  - Control orientations $f_i\mathbf{v}_i \rightarrow \mathbf{f}_i^d$
- Quadrotor Dynamics :

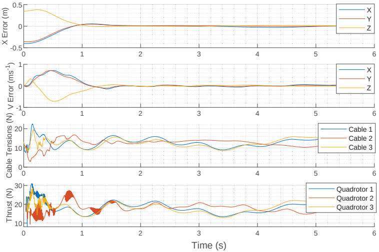
  Translation : $f_i\mathbf{v}_i + m_i\mathbf{g} - t_i\mathbf{u}_i = m_i\ddot{\mathbf{x}}_i$

  Rotation : $\mathbf{m}_i - \mathbf{d} \times t_i\mathbf{u}_i = \mathbb{J}_i\dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times \mathbb{J}_i\boldsymbol{\omega}_i$
- Fix desired yaw $= 0$, $\mathbf{R}^d \in \mathbb{R}^3$

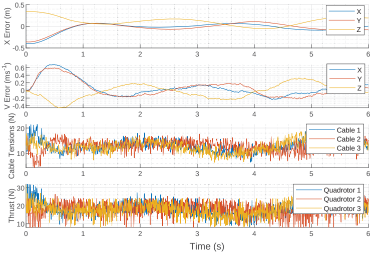
Simulation Results



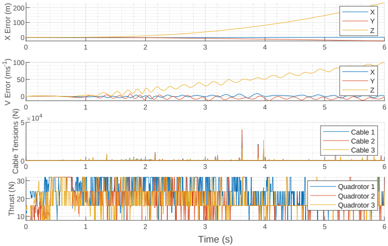
OL rate = 200 Hz, No Noise



OL rate = 50 Hz, No Noise



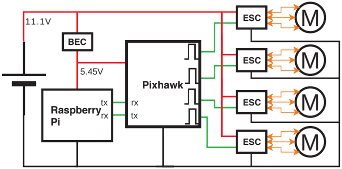
OL rate = 50 Hz, Noise = $\pm 1mm$, $\pm 1^o$

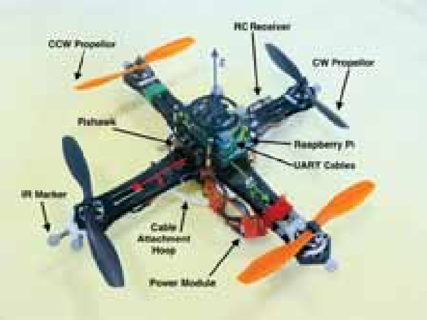


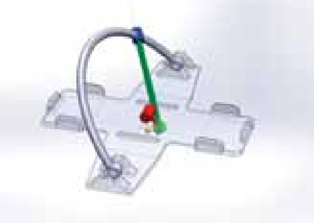
OL rate = 30 Hz, Noise = $\pm 1mm$, $\pm 1^o$

Quadrotors

- Quadrotor Hardware :
  - $m = 1050g$
  - $\bar{f} = 18N$
  - Pixhawk Flight Control Unit
    - Accelerometer
    - Gyroscope
    - Magnemometer
  - Raspberry Pi Computer
- Quadrotor Software
  - RPi - Ubuntu 16.04
  - Pixhawk - NuttX (RTOS)

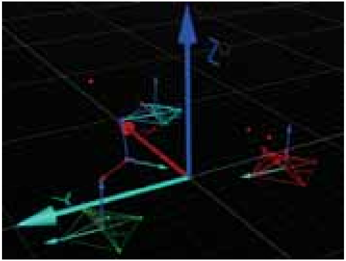


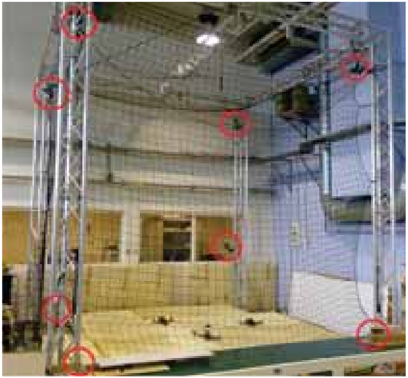




Motion Capture

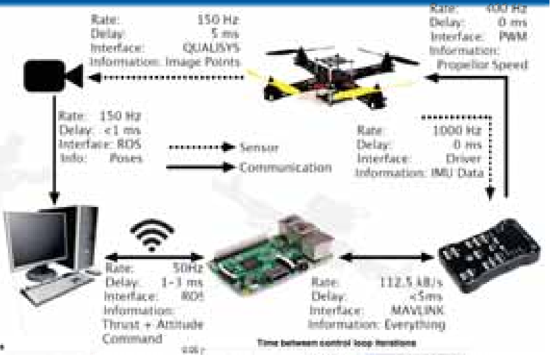
- QUALYSIS System
  - 8 Cameras
  - Passive IR markers
  - 1mm Accuracy
  - $100 - 250Hz$
    - $150Hz$ chosen
  - Latency : $5ms$
  - 6DOF Pose
  - Quadrotors + Payload

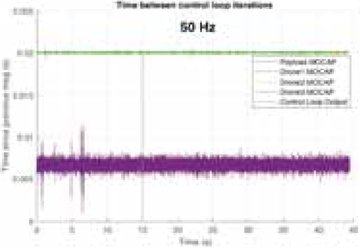


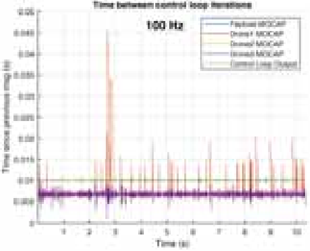


ACTS Design

- Control : Backstepping
  - Outer Loop : ROS over wifi
  - Inner Loop : PX4 embedded
- Communication speeds
  - Qualisys → Computer
  - Computer → RPi
  - RPi → Pixhawk

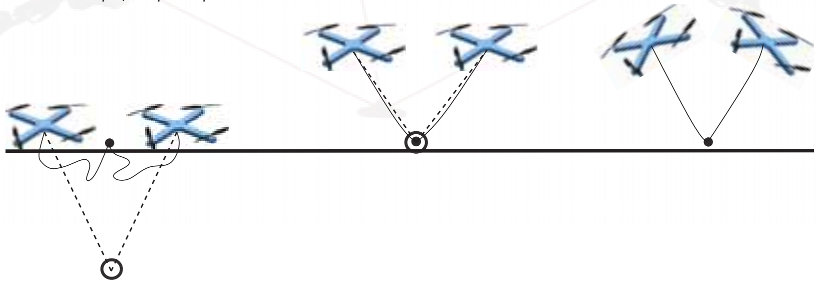






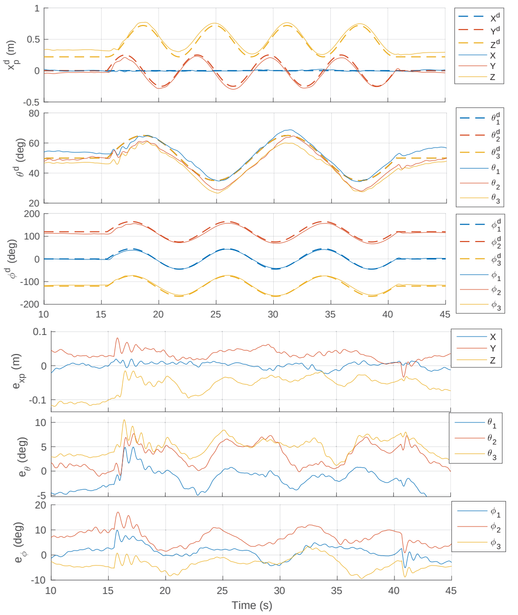
Non-Representative Models

- Takeoff/Landing Problems :
    - Actuation Singularity
    - $t_i \ngtr 0N$
- Solution : Virtual ACTS
    - $\hat{\mathbf{x}}_p = DGM(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$
    - $\mathcal{C} = \mathcal{C}(\hat{\mathbf{x}}_p, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$
    - While $\hat{\mathbf{x}}_p \neq \mathbf{x}_p$, $m_p = 0$

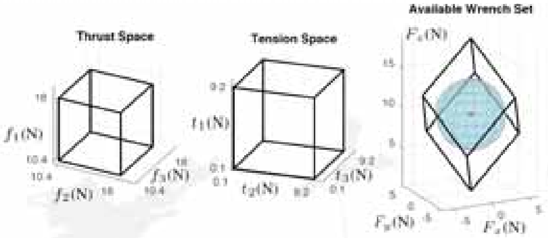


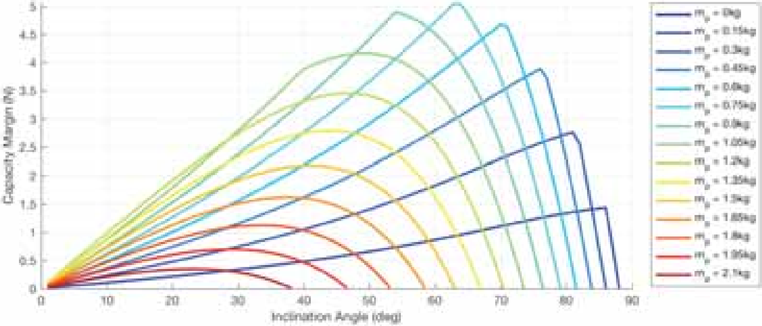
Trajectory Tracking

- Trigonometric trajectories
  - Quick to test
  - Easily differentiable
  - Future : 5^{th} order splines
- RMS position error ≈ 0.08m
  - ≈ +0.05 m bias along z_0
  - +1% mass error



Wrench Analysis

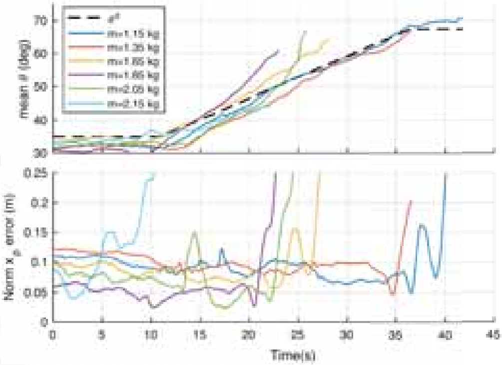




Wrench Analysis - Video

Wrench Analysis - Results

- $\theta_{min} = 35^o$ due to colisions
- Wrench limit validation
  - ACTS limit : 2.15 kg
  - $\gamma = 0$ accurately predicts lose of controllability
  - γ does not affect accuracy
- θ^d not exactly symmetric ($\pm 5^o$)

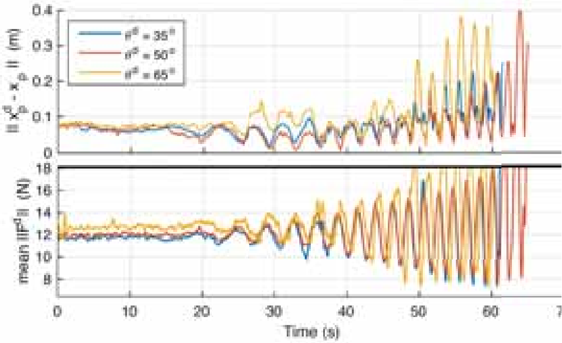


Mass (kg)	1.15	1.35	1.65	1.85	2.05	2.15
$\bar{\gamma}$ (N)	3.7	2.8	1.6	1.0	0.5	0.25
$\theta(\gamma = 0)$ (deg)	71	67	58	51	40	34
θ_{crash} (deg)	70	65	60	55	50	35
Error (deg)	1	2	-2	-4	10	-1

Dynamic Trajectory - Video

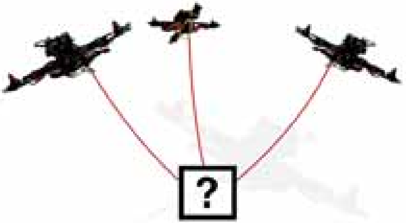
Dynamic Trajectory - Results

- $m_p = 0.65$ kg
- Increase $||\ddot{\mathbf{x}}_p^d||$ until crash
- All configurations crashed between $||\ddot{\mathbf{x}}_p^d|| = 0.8\ \mathrm{ms}^{-2}$ and $||\ddot{\mathbf{x}}_p^d|| = 1.1\ \mathrm{ms}^{-2}$
- Within wrench capabilities of the ACTS
- Possibly unstable internal dynamics



Future Work

- Current Prototype
  - Dynamic wrench analysis
  - Adaptive gain controller
  - Add redundancy



- Real World Deployment
  - Teleoperation $\rightarrow \dot{\mathbf{x}}_p^d, \mathcal{C}^d$
  - Internal $\mathcal{C}$ measurements
- Manipulation Tasks
  - $\mathbf{w}_e(t) \neq$ constant
  - Coupled cable design
  - Robotic end effector

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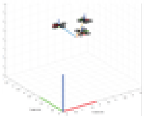
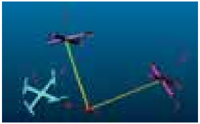
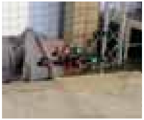
Summary

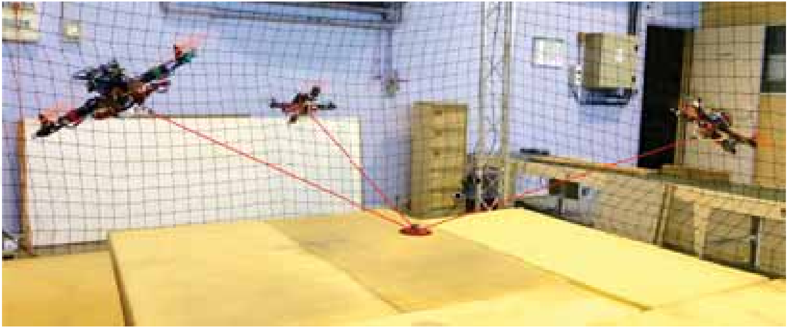
Work Completed :

- Formulate a general task space wrench analysis method
- Developed dynamic controller for a 3-Quadrotor ACTS
- Built and tested a prototype with comparable accuracy to other labs
- Validated configuration limits calculated through wrench analysis

Project Evolution :

 →  →  → 

Thank You



Contact : julian.erskine@ls2n.fr

Matrices

$$
\mathbf{J} = \begin{bmatrix}
1 & 0 & 0 & -l_1 s\phi_1 s\theta_1 & l_1 c\phi_1 c\theta_1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & l_1 c\phi_1 s\theta_1 & l_1 s\phi_1 c\theta_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -l_1 s\theta_1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -l_2 s\phi_2 s\theta_2 & l_2 c\phi_i c\theta_2 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & l_2 c\phi_2 s\theta_2 & l_2 s\phi_2 c\theta_2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -l_2 s\theta_2 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -l_3 s\phi_3 s\theta_3 & l_3 c\phi_3 c\theta_3 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & l_3 c\phi_3 s\theta_3 & l_3 s\phi_3 c\theta_3 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -l_3 s\theta_3
\end{bmatrix}^{-1}
$$

$$
\mathbf{b} = \begin{bmatrix}
-l_1 \begin{bmatrix} c\phi_1 s\theta_1 & c\phi_1 s\theta_1 & 2s\phi_1 c\theta_1 \\ s\phi_1 s\theta_1 & s\phi_1 s\theta_1 & 0 \\ 0 & c\theta_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1^2 \\ \dot{\theta}_1^2 \\ \dot{\phi}_1 \dot{\theta}_1 \end{bmatrix} \\
-l_2 \begin{bmatrix} c\phi_2 s\theta_2 & c\phi_2 s\theta_2 & 2s\phi_2 c\theta_2 \\ s\phi_2 s\theta_2 & s\phi_2 s\theta_2 & 0 \\ 0 & c\theta_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_2^2 \\ \dot{\theta}_2^2 \\ \dot{\phi}_2 \dot{\theta}_2 \end{bmatrix} \\
-l_3 \begin{bmatrix} c\phi_3 s\theta_3 & c\phi_3 s\theta_3 & 2s\phi_3 c\theta_3 \\ s\phi_3 s\theta_3 & s\phi_3 s\theta_3 & 0 \\ 0 & c\theta_3 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_3^2 \\ \dot{\theta}_3^2 \\ \dot{\phi}_3 \dot{\theta}_3 \end{bmatrix}
\end{bmatrix}
$$

Matrices

$$\mathbf{f} = \underbrace{\left(\mathbf{M}_Q\mathbf{J}^{-1} + \mathbf{T}_{\ddot{x}_p}\right)}_{\mathbf{D}}\ddot{\mathbf{X}} - \underbrace{\mathbf{M}_Q\left(\begin{bmatrix}\mathbf{g}\\\mathbf{g}\\\mathbf{g}\end{bmatrix} + \mathbf{J}^{-1}\mathbf{b}\right) - \mathbf{T}_g}_{\mathbf{G}}$$

$$\mathbf{M}_Q = \begin{bmatrix} m_1\mathbb{I}_3 & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & m_2\mathbb{I}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & m_3\mathbb{I}_3 \end{bmatrix}$$

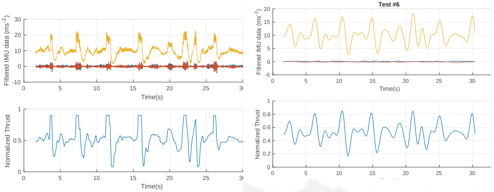
$$\mathbf{T}_{\ddot{x}_p} = m_p\begin{bmatrix} \mathbf{u}_1\boldsymbol{i}\mathbf{W}^{-1}, & \mathbf{0}_{3\times6} \\ \mathbf{u}_2\boldsymbol{j}\mathbf{W}^{-1}, & \mathbf{0}_{3\times6} \\ \mathbf{u}_3\boldsymbol{k}\mathbf{W}^{-1}, & \mathbf{0}_{3\times6} \end{bmatrix}$$

$$\mathbf{T}_g = \begin{bmatrix} \mathbf{u}_1\boldsymbol{i}\mathbf{W}^{-1}(m_p\mathbf{g} + \mathbf{w}_e) \\ \mathbf{u}_2\boldsymbol{j}\mathbf{W}^{-1}(m_p\mathbf{g} + \mathbf{w}_e) \\ \mathbf{u}_3\boldsymbol{k}\mathbf{W}^{-1}(m_p\mathbf{g} + \mathbf{w}_e) \end{bmatrix}$$

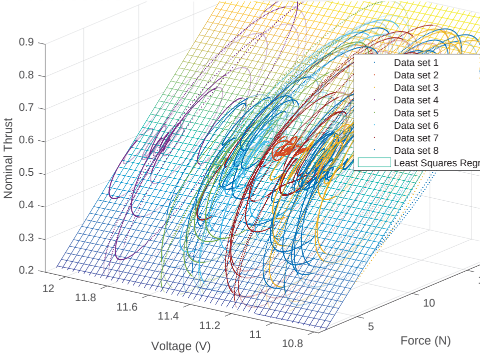
$\boldsymbol{i} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$, $\boldsymbol{j} = \begin{bmatrix}0 & 1 & 0\end{bmatrix}$, and $\boldsymbol{k} = \begin{bmatrix}0 & 0 & 1\end{bmatrix}$.

Quadrotors

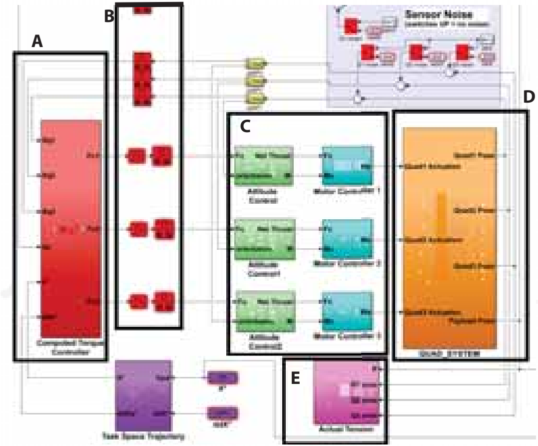
- Controller Thrust : $f \in \mathbb{R}$
- PX4 Thrust : $n = [0 - 1]$ (Saturated at 0.9)
- Need mapping $f \rightarrow n$
  - Determine actual thrust : $\mathbf{R}_i f_i \mathbf{z}_0 = m_i \mathbf{R}_i \ddot{\mathbf{x}}_{i,imu}$
  - Empirical : $n = af + bV + c$



$$n = 0.04f - 0.049V + 0.65$$



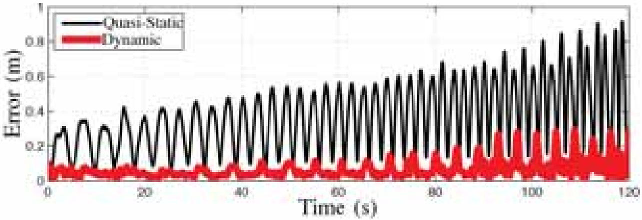
Simulation



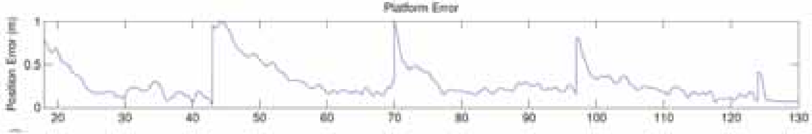
A : Outer Control Loop
B : Delays and Rate Change
C : Attitude Control Loop
D : Plant Model
E : Check Real Tensions

Existing Controllers

- Differential Flatness : $\mathbf{X} = \left[\mathbf{x}_p, t_2\mathbf{u}_2, t_3\mathbf{u}_3\right]$



- LQR : good for avoiding collisions



- Geometric Controller Controller
  - $\mathbf{X} = \left[\mathbf{x}_p, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right]$
  - Only simulations

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