The True Role of Accelerometer Feedback in Quadrotor Control

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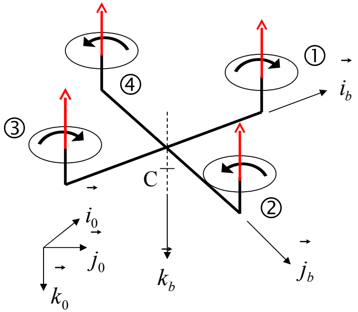
2010 IEEE Int. Conf. on Robotics and Automation

Two puzzling questions

Quadrotor:

- 2 CW, 2 CCW props
- $(\vec{\imath}_b, \vec{\jmath}_b, \vec{k}_b)$ body axes



Newton's law for usual model

$$\dot{\vec{V}}_C = \vec{g} + \frac{T}{m}\vec{k}_b$$

Triaxial accelero located at C

$$\vec{a} = \dot{\vec{V}}_C - \vec{g} \quad \text{(in body axes)}$$

This implies $\vec{a} = \frac{T}{m}\vec{k}_b$, hence

$$(a_x, a_y) = (\vec{a}\cdot\vec{\imath}_b, \vec{a}\cdot\vec{\jmath}_b) = (0,0)$$

What is accelero feedback good for?

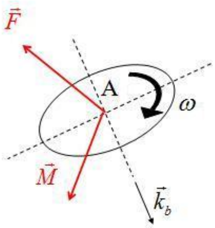
Usual approximation: $\vec{a} \approx -\vec{g}$, hence

$$(a_x, a_y) \approx (g\sin\theta, -g\sin\phi\cos\theta)$$

Meaning of the approximation?

Single propeller

- angular velocity $\varepsilon\omega$ around axis $\vec{k}_b$
  $\omega \geq 0$, $\varepsilon = \pm 1$ (CCW/CW)
- $\vec{V}_A$ linear velocity of prop center A
- $\vec{\Omega}$ angular velocity of "rotor plane" $\perp \vec{k}_b$



Aerodynamic efforts "near" hovering

$$\vec{F} = -a\omega^2\vec{k}_b - \omega\left(\lambda_1 \vec{V}_A^\perp - \lambda_2\vec{\Omega} \times \vec{k}_b\right) + \varepsilon\omega\left(\lambda_3 \vec{V}_A \times \vec{k}_b - \lambda_4\vec{\Omega}^\perp\right)$$

$$\vec{M} = -b\varepsilon\omega^2\vec{k}_b - \omega\left(\mu_1 \vec{V}_A^\perp + \mu_2\vec{\Omega} \times \vec{k}_b\right) - \varepsilon\omega\left(\mu_3 \vec{V}_A \times \vec{k}_b + \mu_4\vec{\Omega}^\perp\right)$$

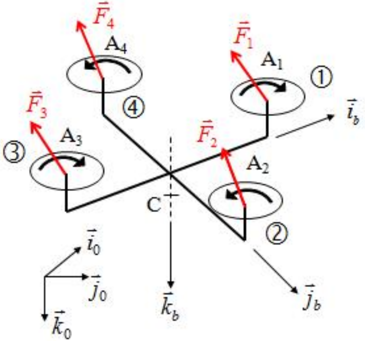
$a, b, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \mu_1, \mu_2, \mu_3, \mu_4$ positive constants
projection on rotor plane $\vec{U}^\perp := \vec{k}_b \times (\vec{U} \times \vec{k}_b) = \vec{U} - (\vec{U} \cdot \vec{k}_b)\vec{k}_b$

Classical blade element theory, with two extra simplifications:

- higher-order linear and angular velocity terms neglected
- linear and angular accelerations neglected

Complete quadrotor:

- quadrotor $\mathcal{B}$ = frame $\mathcal{B}_0$ + 4 $\times$ (prop + motor) $\mathcal{B}_i$
- 2 CW, 2 CCW props
- $(\vec{\imath}_b, \vec{\jmath}_b, \vec{k}_b)$ body axes



Motion equations (dimension 13)

$$\dot{\vec{V}}_C = \vec{g} + \frac{1}{m}\sum_{i=1}^{4}\vec{F}_i$$

$$\dot{\vec{\sigma}}_C^{\mathcal{B}} = \sum_{i=1}^{4}\vec{CA}_i \times \vec{F}_i + \vec{M}_i$$

$$\dot{\vec{\sigma}}_{A_i}^{\mathcal{B}_i} \cdot \vec{k}_b = \vec{M}_i \cdot \vec{k}_b + \varepsilon_i \Gamma_i, \;\; i = 1, 2, 3, 4$$

$$\dot{R}_{\phi,\theta,\psi} = \vec{\Omega} \times R_{\phi,\theta,\psi}$$

Inertial sensors (in body axes)

accelero: $\vec{a} = \dot{\vec{V}}_C - \vec{g} = \frac{1}{m}\sum_{i=1}^{4}\vec{F}_i$

gyro: $\vec{\Omega}$

with some simplifications (near hovering and mostly rigid prop)

$$\dot{\vec{V}}_C \approx \vec{g} - \frac{a}{m}\Big(\sum_{i=1}^{4}\omega_i^2\Big)\vec{k}_b - \frac{\lambda_1}{m}\Big(\sum_{i=1}^{4}\omega_i\Big)\vec{V}_C^{\perp}$$

$$\dot{\vec{\sigma}}_C^{\mathcal{B}} \approx al(\omega_4^2 - \omega_2^2)\vec{\imath}_b + al(\omega_1^2 - \omega_3^2)\vec{\jmath}_b - b\Big(\sum_{i=1}^{4}\varepsilon_i\omega_i^2\Big)\vec{k}_b - \Big(\sum_{i=1}^{4}\omega_i\Big)\big(\lambda_1 l^2 r\vec{k}_b + \mu_1'\vec{V}_C \times \vec{k}_b + \mu_2''\vec{\Omega}^{\perp}\big)$$

$$\dot{\vec{\sigma}}_{A_i}^{\mathcal{B}_i}\cdot\vec{k}_b \approx \varepsilon_i\Gamma_i - \Big(\sum_{i=1}^{4}\omega_i\Big)\lambda_1 l^2 r, \qquad i = 1,2,3,4$$

$$\dot{R}_{\phi,\theta,\psi} = \vec{\Omega}\times R_{\phi,\theta,\psi}$$

$$\vec{a} \approx -\frac{a}{m}\Big(\sum_{i=1}^{4}\omega_i^2\Big)\vec{k}_b - \frac{\lambda_1}{m}\Big(\sum_{i=1}^{4}\omega_i\Big)\vec{V}_C^{\perp}$$

Linearized model splits into four independent subsystems:

- longitudinal (input $\Gamma_q := \frac{\Gamma_1 - \Gamma_3}{J_r}$, states $u, \theta, q, \omega_q := \omega_1 - \omega_3$)

$$\dot{u} = -g\theta - f_1 u \qquad \text{(longitudinal velocity)}$$
$$\dot{\theta} = q \qquad \text{(pitch angle)}$$
$$\dot{q} = f_4\omega_q + f_2 u - f_3 q \qquad \text{(pitch rate)}$$
$$\dot{\omega}_q = \Gamma_q - f_5\omega_q \qquad \text{(prop difference)}$$

  Measurements $a_x = -f_1 u$ and $g_y = q$
- lateral (input $\Gamma_4 - \Gamma_2$, states $v, \phi, p, \omega_4 - \omega_2$)
- vertical (input $\sum_{i=1}^4 \Gamma_i$, states $w, \sum_{i=1}^4 \omega_i$)
- heading (input $\sum_{i=1}^4 \varepsilon_i \Gamma_i$, states $\psi, r, \sum_{i=1}^4 \varepsilon_i \omega_i$)

$$(f_1, f_2, f_3, f_4, f_5) := \left(\frac{4\lambda_1\bar{\omega}}{m}, \frac{4\mu_1'\bar{\omega}}{I}, \frac{4\mu_2''\bar{\omega}}{I}, \frac{2al\bar{\omega}}{I}, \frac{2b\bar{\omega}}{J_r}\right)$$

Experimental setup: home-built “Quadricopter” + radio data link

- “true” Earth velocity V_x, V_y, V_z and orientation ϕ_m, θ_m, ψ_m given by MIDG2 “GPS-aided Inertial Navigation System”
- raw accelero data a_{xm}, a_{ym} also given by MIDG2
- quadrotor flown in back and forth translations for 1 minute
- seek to validate force model only (because of low throughput of radio data link)

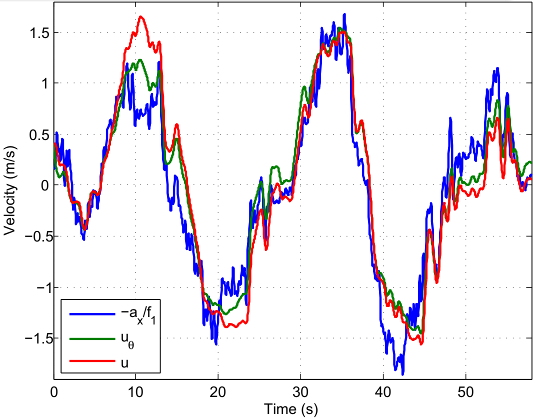
Processing of flight data

- filter all the data with 5^{th}-order Bessel @ 2Hz, which preserves the transfer functions
- compensate data for MIDG2 misalignement ϕ_0, θ_0, ψ_0
- compute “true” body velocities u, v, w
- compute aligned accelero measurements a_x, a_y

Validation of force model

With $(\frac{1}{f_1}, \phi_0, \theta_0, \psi_0) := (4\text{s}, 1.2^\circ, -2.4^\circ, 2^\circ)$ good fit between:

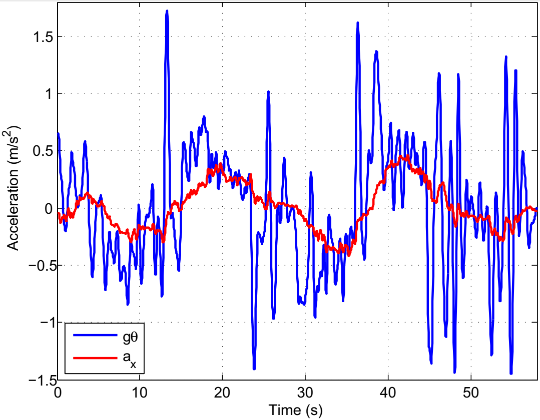
- "true" body velocity u
- "accelerometer-based" velocity $-\frac{a_x}{f_1}$
- velocity u_θ "predicted" by model from "true" pitch angle θ



Invalidation of no-acceleration assumption $\vec{a} \approx -\vec{g}$

$$a_x \approx g \sin\theta \quad ???$$

- more or less ok in average... because quadrotor stabilized by other means
- very wrong transients!



Importance of coefficients wrt "full-state" feedback?

$$\dot{u} = -g\theta - f_1 u$$
$$\dot{\theta} = q$$
$$\dot{q} = f_4\omega_q + f_2 u - f_3 q$$
$$\dot{\omega}_q = \Gamma_q - f_5\omega_q$$

$$\Gamma_q = -\frac{k_p}{\varepsilon^2} q - \frac{k_d}{\varepsilon}\dot{q} + \frac{k_p}{\varepsilon^2} q_r \quad \text{(inner loop)}$$
$$q_r = k_1 u + k_2 \dot{u} - k_1 u_r \quad \text{(outer loop)}$$

After fast inner loop:

$$\dot{u} = -f_1 u - g\theta$$
$$\dot{\theta} = q$$
$$\varepsilon\dot{q} = f_4\tilde{\omega}_q + \mathcal{O}(\varepsilon)$$
$$\varepsilon\dot{\tilde{\omega}}_q = -k_p q - f_4 k_d \tilde{\omega}_q + k_p q_r + \mathcal{O}(\varepsilon)$$

Singular perturbations ($\tilde{\omega}_q := \varepsilon\omega_q$)
$\Rightarrow$ f_2, f_3, f_5 dominated by feedback

After slow outer loop:

$$\dot{u} = -f_1 u - g\theta$$
$$\dot{\theta} = (k_1 - f_1 k_2)u - gk_2\theta - k_1 u_r$$

Reasonable settling time
$\Rightarrow$ f_1 dominated by feedback

Conclusion: if u (and q) measured, revisited state model not useful!

Slow control model

$$\dot{u} = -g\theta - f_1 u$$

$$\dot{\theta} \approx q_r \quad \text{(for controller)}$$

$$\dot{\theta} = q \quad \text{(for estimator)}$$

Outer loop (with "angle estimator")

$$q_r = k(\theta_r - \hat{\theta})$$

$$\dot{\hat{\theta}} = q + l\Big(\frac{a_x}{g} - \hat{\theta}\Big)$$

Usual interpretation

$a_x = g\theta \Rightarrow$ closed-loop polynomial $s(s+k)(s+l)$

$$\theta = \frac{k}{s+k}\theta_r$$

$$u = \frac{-gk}{s(s+k)}\theta_r$$

With $k, l > 0$, $(u, \theta, \hat{\theta}) \to (\infty, \theta_r, \theta_r)$

Not quite consistent with experience!

Revisited interpretation

$a_x = -f_1 u \Rightarrow$ closed-loop polynomial

$$\Delta = s^3 + (k + l + f_1)s^2 + f_1(k + l)s + f_1 kl$$

$$\approx (s + k)(s^2 + f_1 s + f_1 l) \quad \text{with } l \ll k$$

$$\approx (s + f_1)(s + k)(s + l) \quad \text{with } l \ll f_1$$

$$\theta = \frac{k(s + f_1)(s + l)}{\Delta}\theta_r \approx \frac{k}{s+k}\theta_r$$

$$u = \frac{-gk(s + l)}{\Delta}\theta_r \approx \frac{-gk}{(s + f_1)(s + k)}\theta_r$$

With $k, l > 0$, $(u, \theta, \hat{\theta}) \to (-\frac{g}{f_1}\theta_r, \theta_r, \theta_r)$

Improve performance with controller based on revisited model

Performance of usual control scheme limited by time constant $\frac{1}{f_1}$
$\Rightarrow$ better performance with eg "revisited controller-observer"

$$q_r = -k_1 \hat{u} - k_2 \hat{\theta} + \left(k_1 - \frac{f_1 k_2}{g}\right) u_r$$

$$\dot{\hat{u}} = -f_1 \hat{u} - g\hat{\theta} + l_1(a_x + f_1 \hat{u})$$

$$\dot{\hat{\theta}} = g_y + l_2(a_x + f_1 \hat{u})$$