# Locomotion Dynamics Modeling
## (Application to bioinspired robotics)

**Authors:** F. Boyer
**Laboratory:** IRCCyN
**Venu:** ENSAM (Paris)
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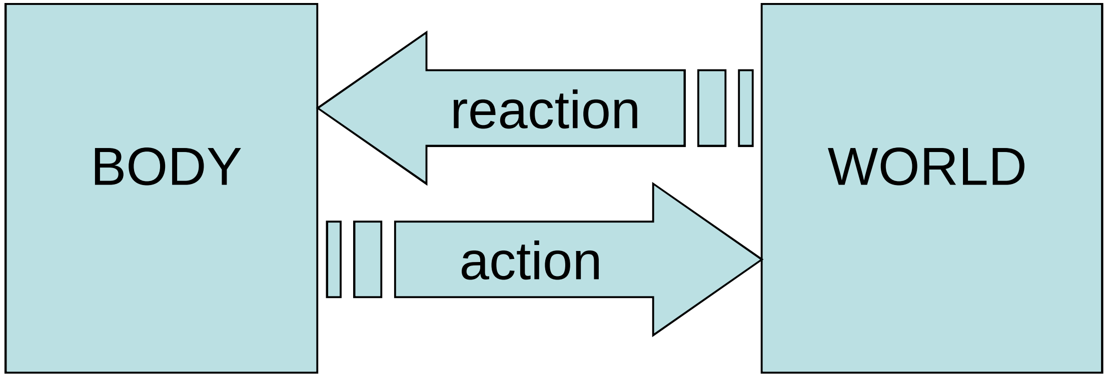
I Introduction: mathematical framework

II Locomotion model

III Applications to bio-inspired (swimming, flying, creeping)

In general locomotion is based on the action-reaction principle




- Endo skeleton : vertebrates
- Exo skeleton : arthropodes
- Soft bodies : mollusques…

- Fluid: underwater / aerial
- Solid : terrestrial locomotion
- Exotic media…
- Void

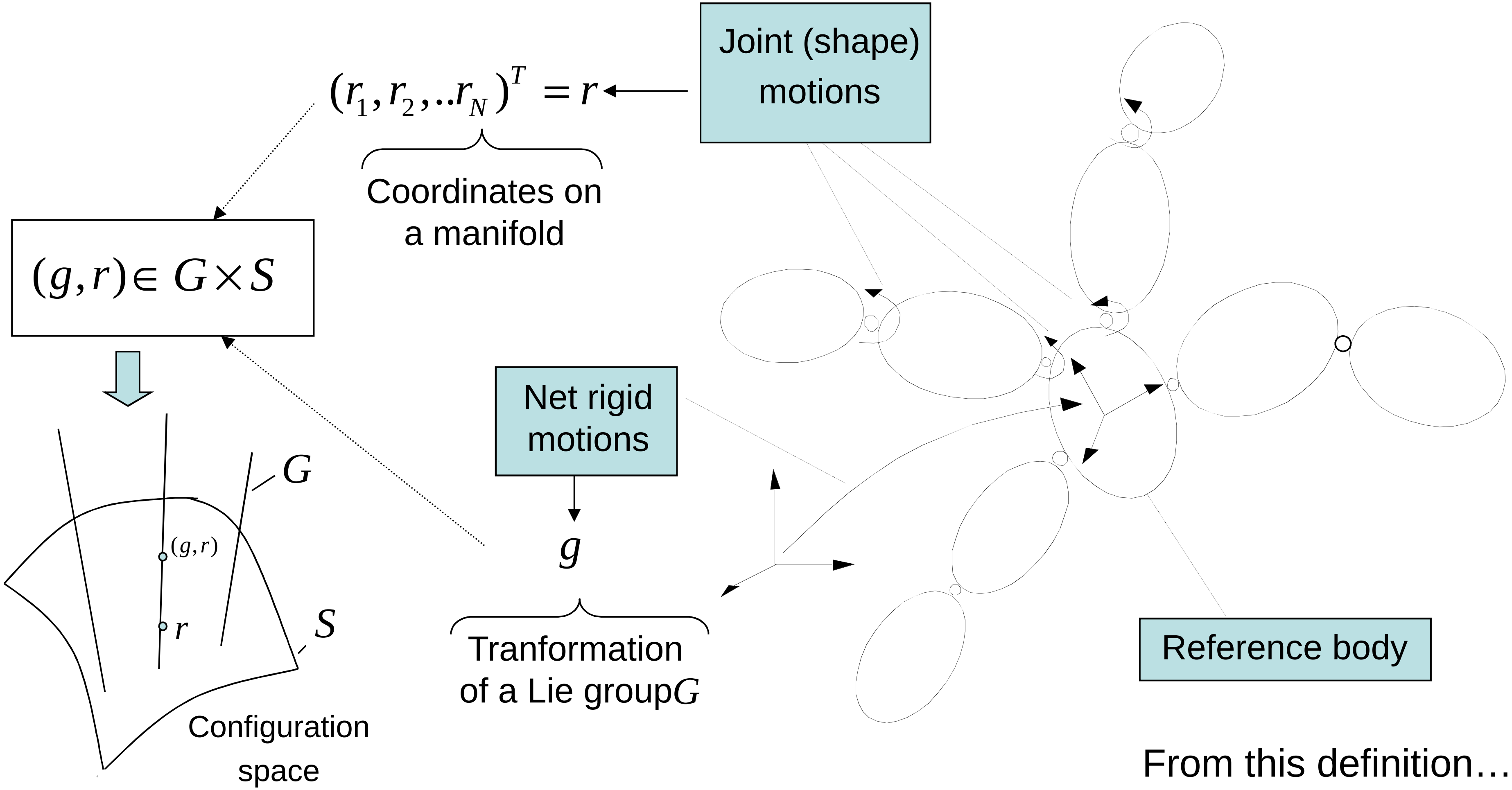
Here we model the animal or robot as a mobile multibody system (MMS)

➡ And ask two general questions:

- A theoretical one : « How can we classify the locomotion models? »
- A practical one : « How efficiently compute these models ? »

Before to try to answer… definition of a MMS

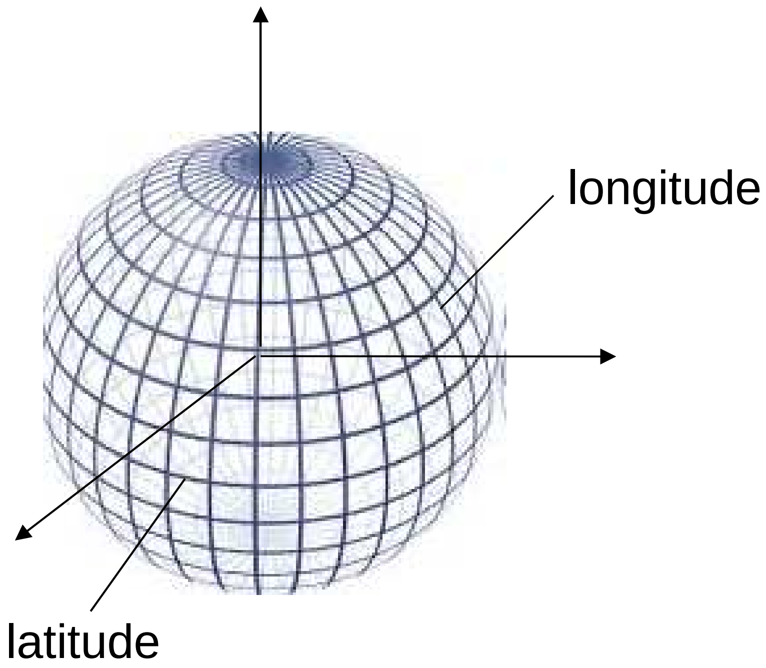
# Definition of a Mobile Multi Body System…




From this definition…

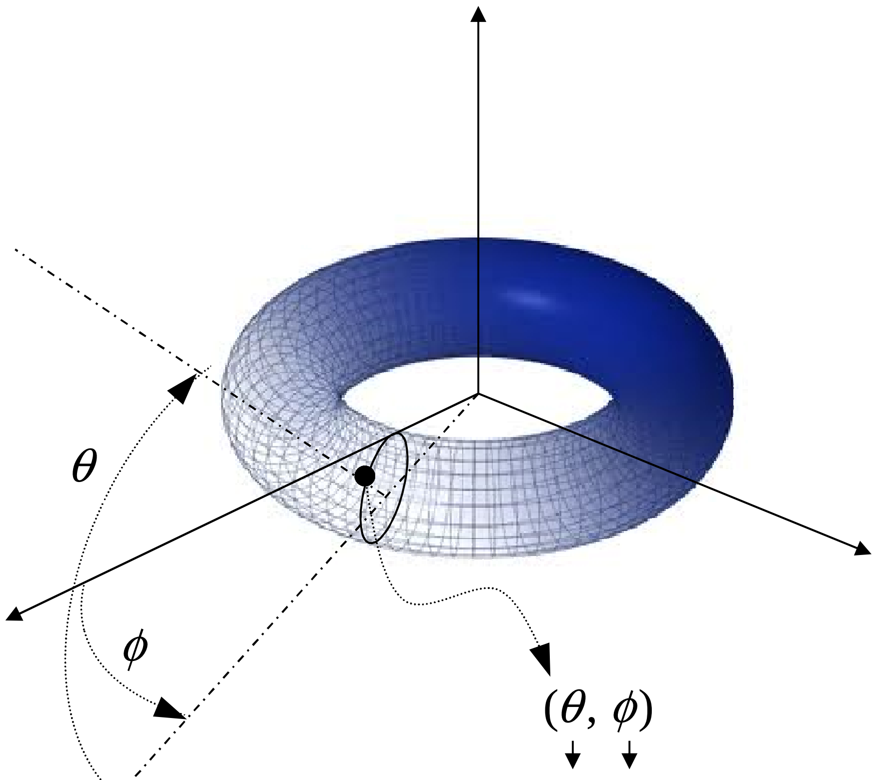
# What is a manifold…?

It's a set of points whose relative positions are known from coordinates charts…




Sphere : $M = S^2$

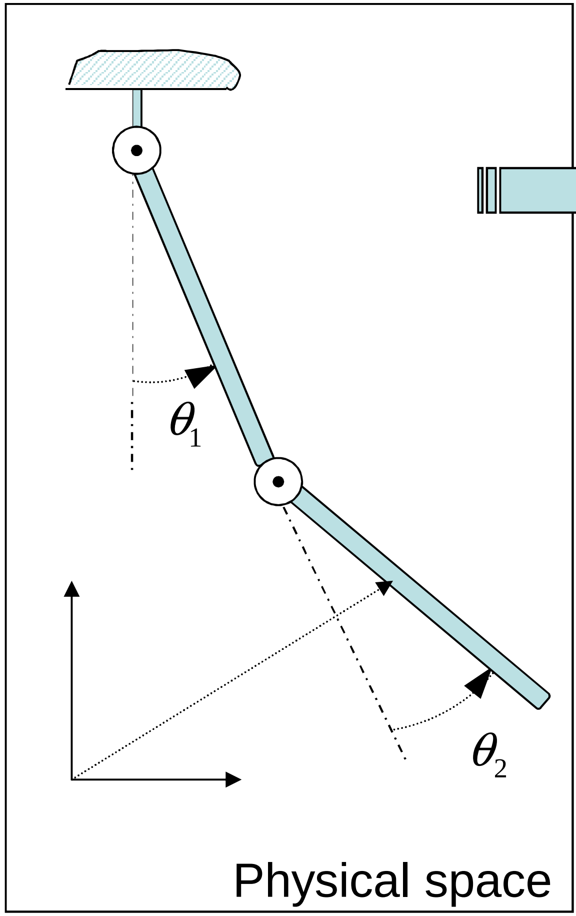
(longitude, latitude) = $(\varphi, \psi)$




Torus : $M = S^1 \times S^1$

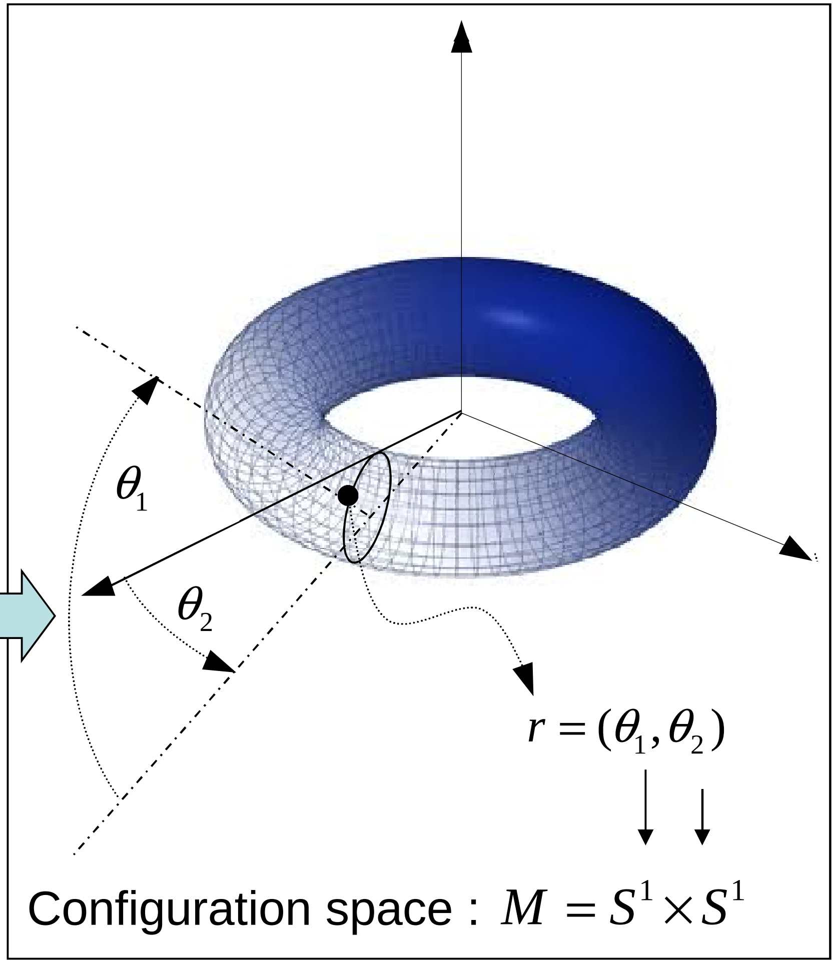
# In mechanics, the motion of a MS is a point moving on a Manifold

Example of the double pendulum…



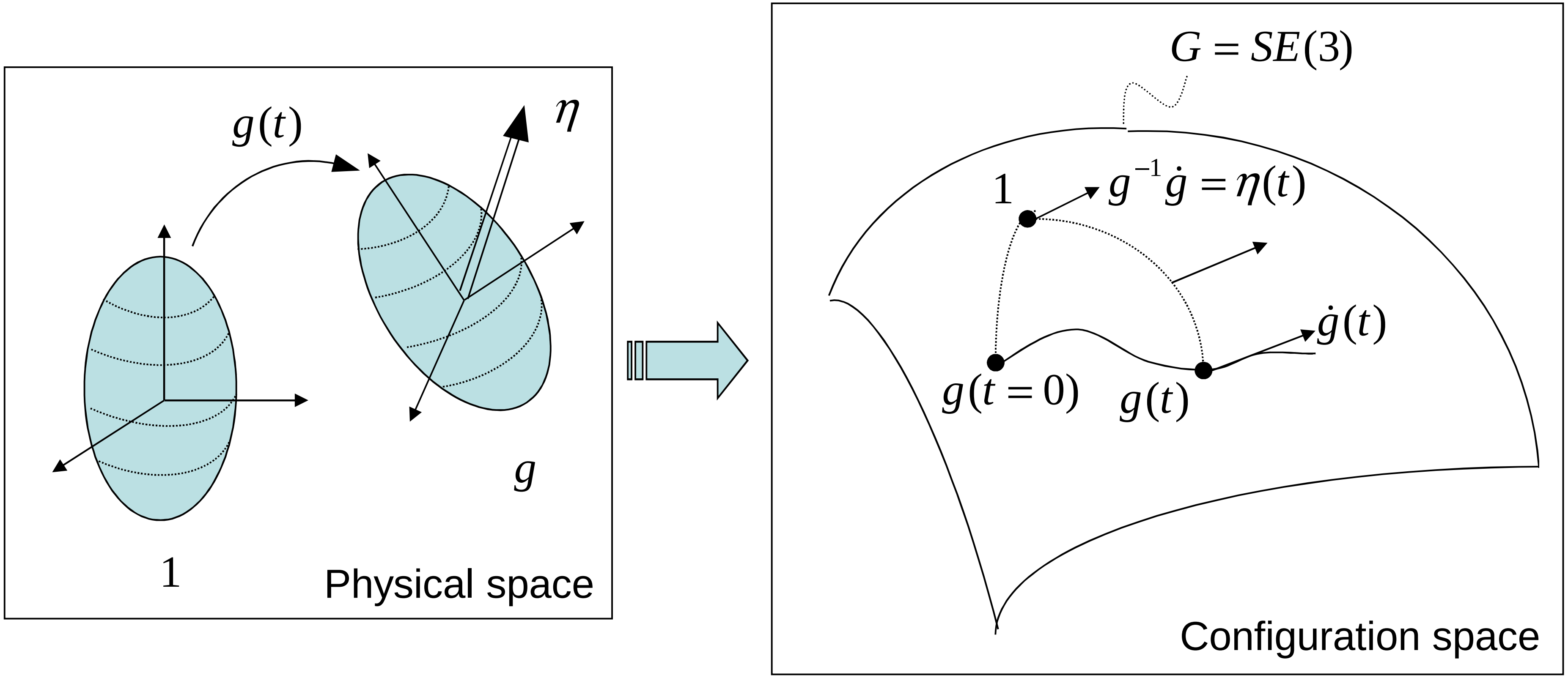

Picture = Configuration In Physical space

Position of one point moving on the 2D torus



# What is a Lie group…?

Starting from an example: the rigid (reference) body…



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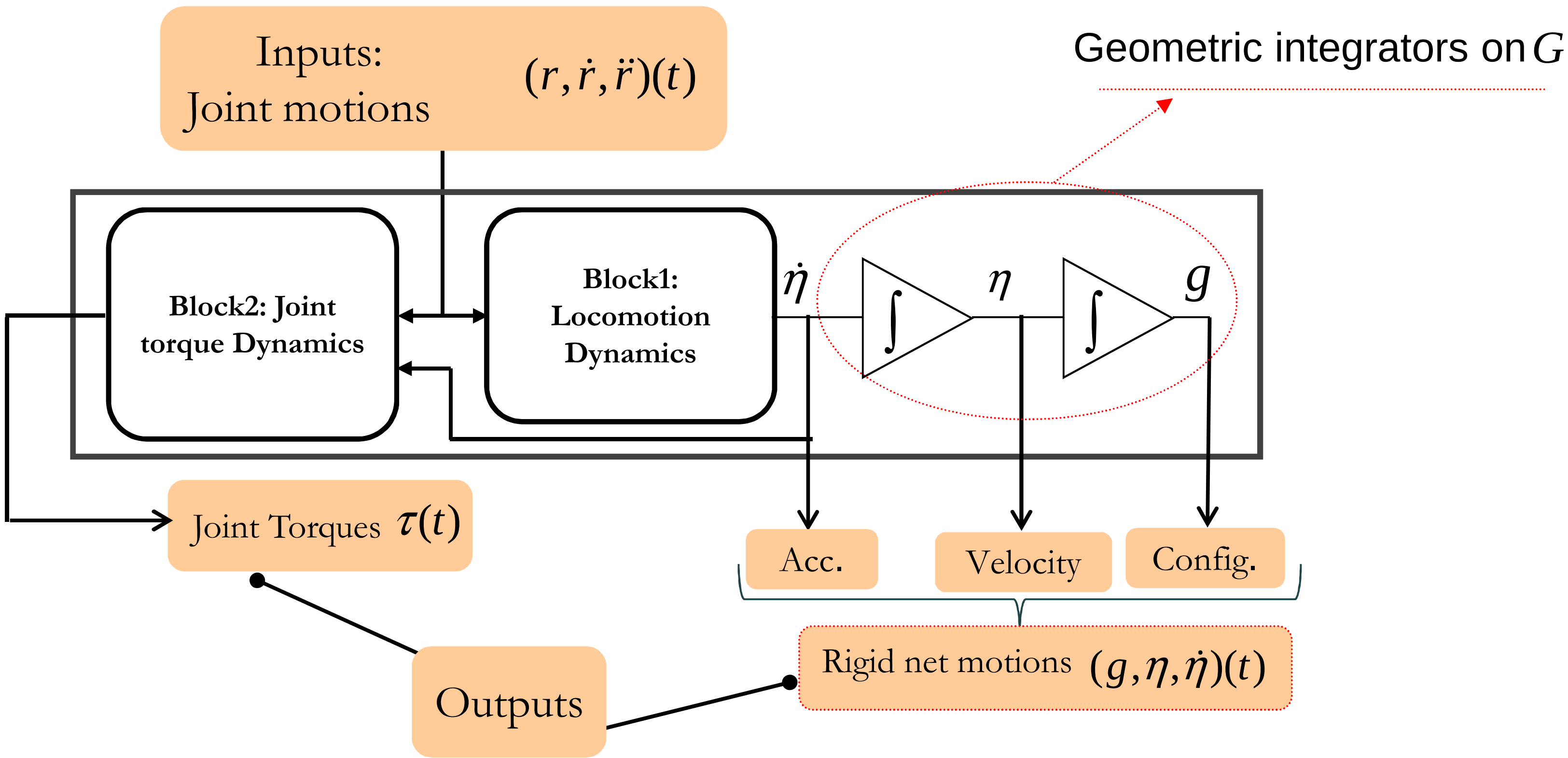
# General problem of locomotion dynamics…

1°) Knowing the joint motions (gaits when they are c yclic, transient maneuvers…)

2°) To compute:

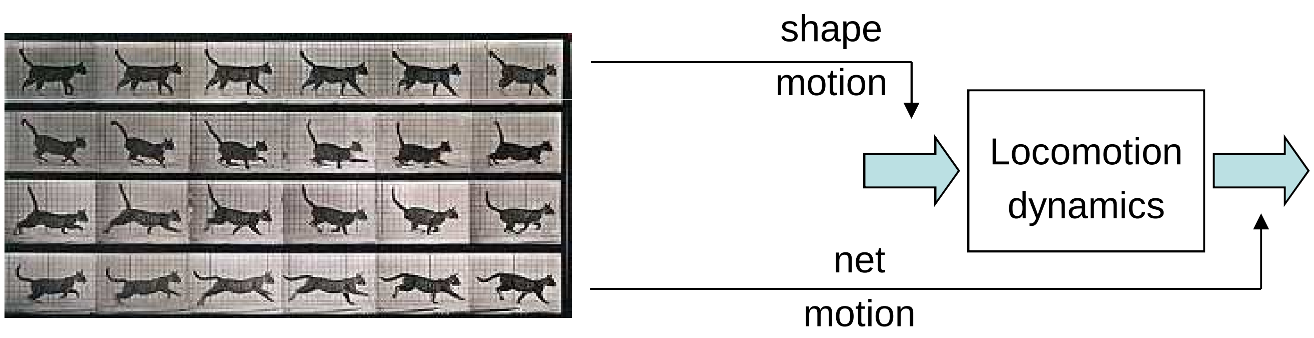
- the net rigid motions : (forward) « locomotion dynamics »
- the joint torques : (inverse) « torque dynamics »

# To solve this pb : General dynamic algorithm for locomotion



- Why this choice, why not take the joint torques as inputs?

More intuitive, … can be coupled to biological experiments based on films…

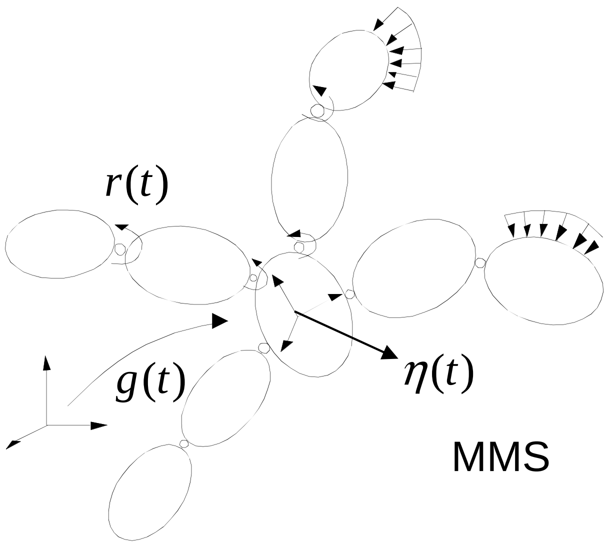



- Another relevent problem for locomotion:

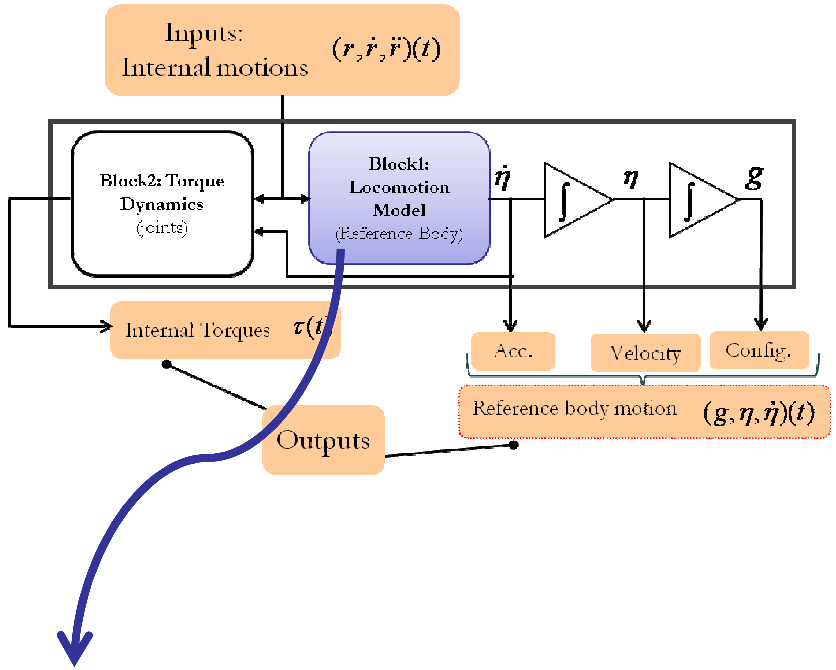
The inverse locomotion problem:

Find the joint motions ensuring a given net motion…

# Forward locomotion dynamics

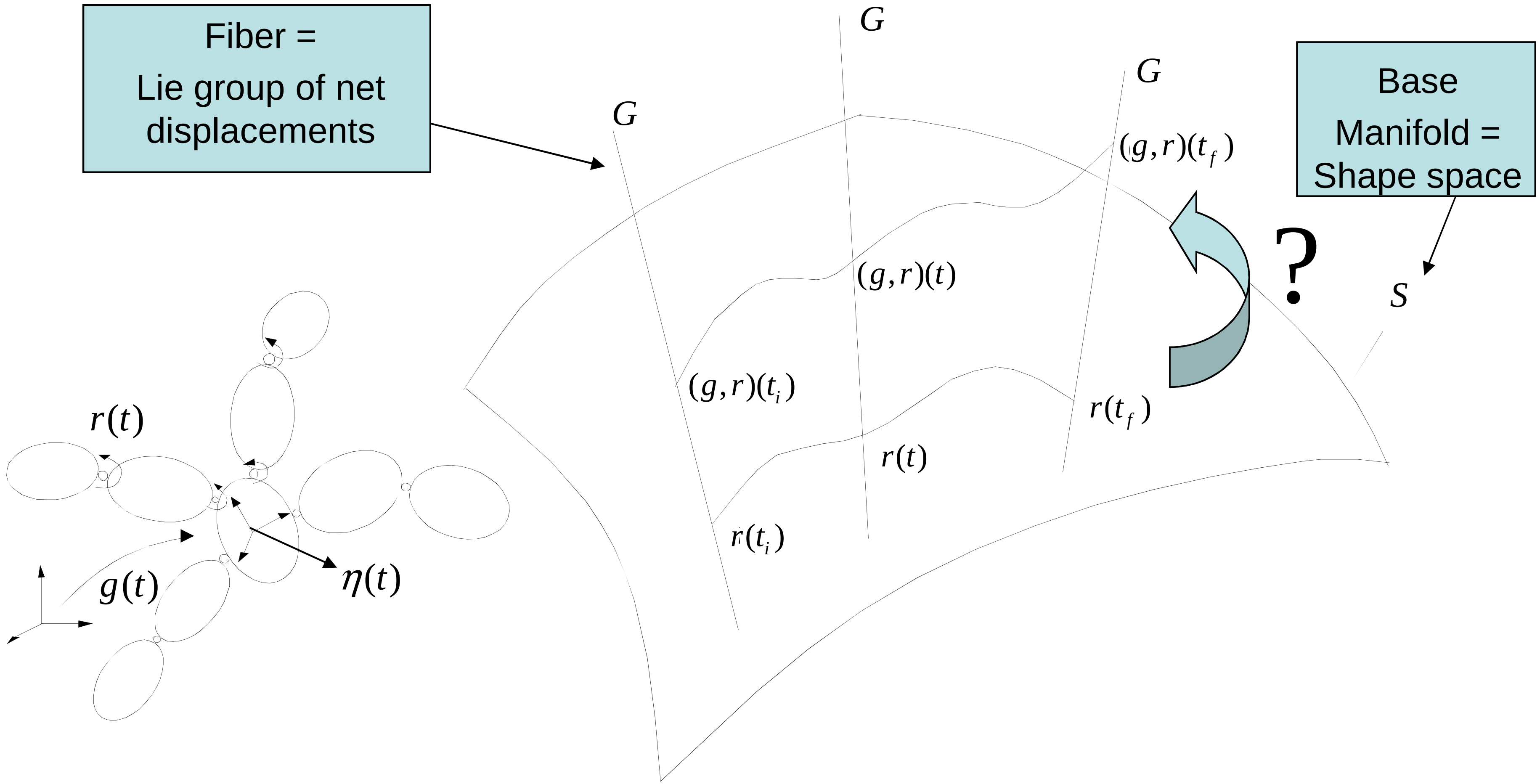



MMS




Joint Imposed motions → **Block1: Locomotion Model** → Net rigid acceleration

On the « principal fiber bundle » of configurations, we seek the link …



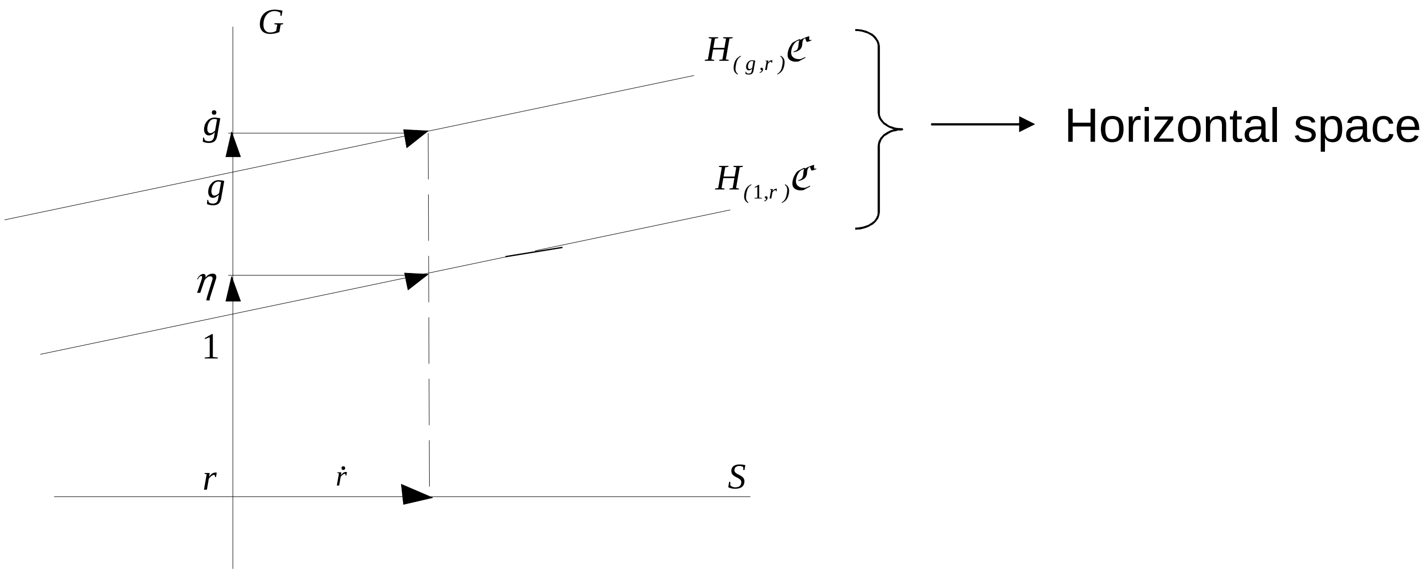
The most simple way of relating $S$ and $G$ : define a connexion, i.e.:

Linear relation between small displacement on $S$ and $G$

Displacements in $G$ independent of $g$ (left invariance)

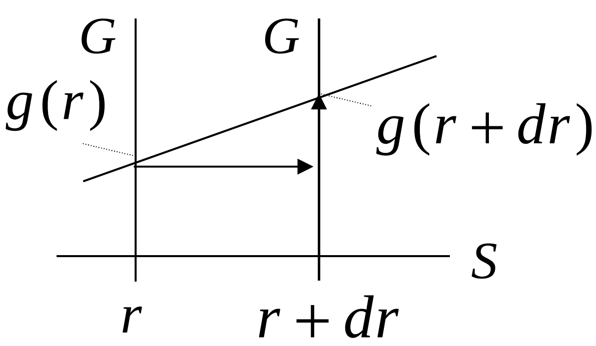
$$\Rightarrow \quad (\dot{g},\dot{r}) \in H_{(g,r)}\mathcal{C} \subset T_{(g,r)}\mathcal{C} \quad \Rightarrow \quad \eta + \mathcal{A}(r)\dot{r} = 0$$

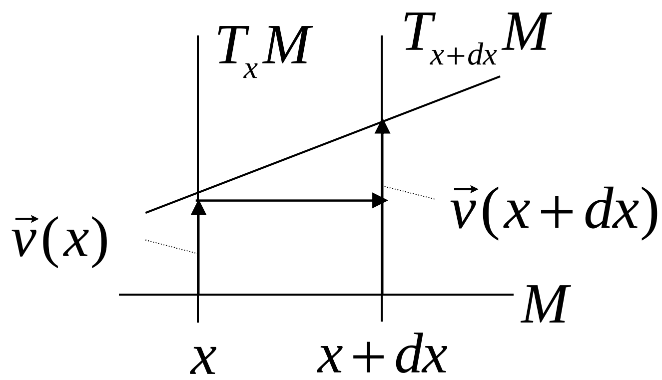
Local connexion



In a more general way…

A connection associates univoqually an fiber element over a point (of the base manifold) to another element of the fiber over a point infinitesimally close…

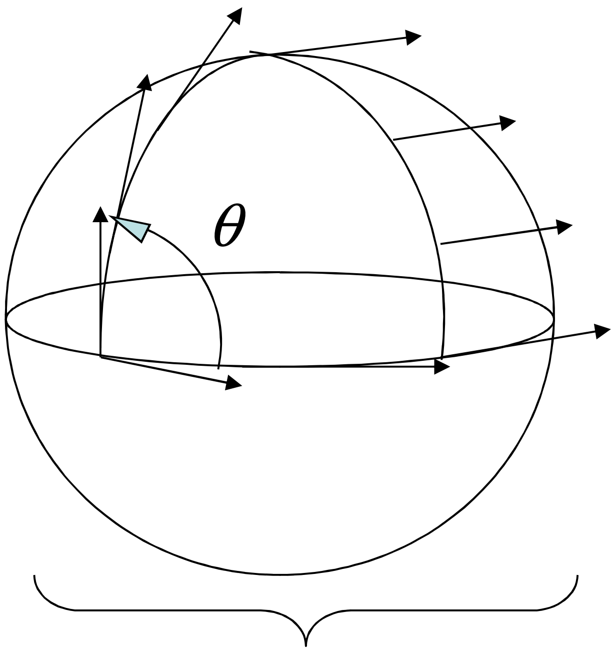


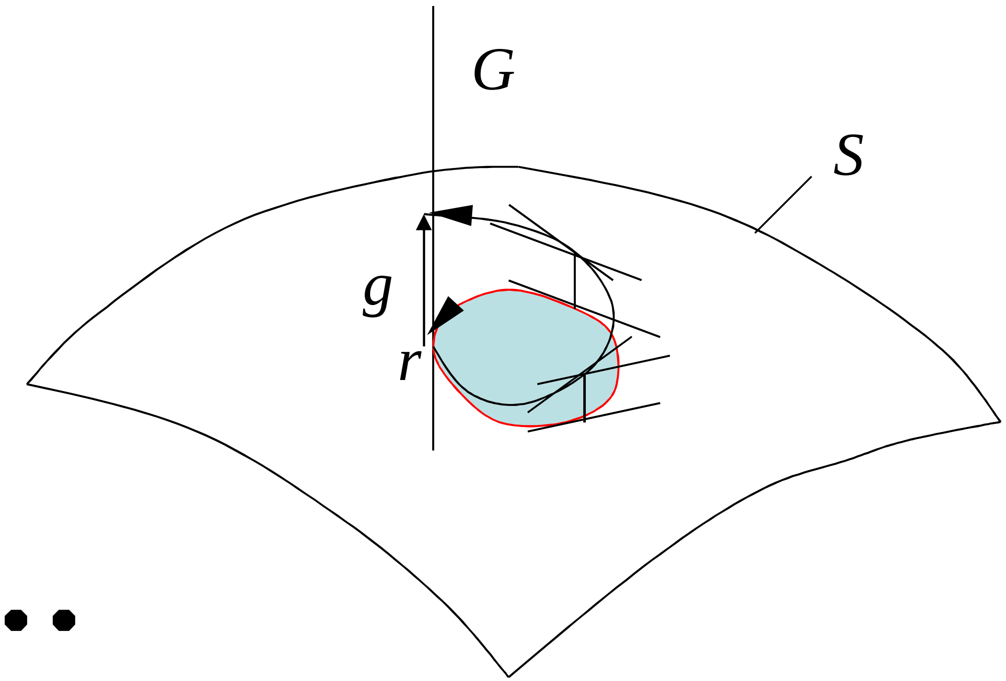

Fiber bundle = $G \times S$ ⇨ Fiber bundle = Tangent bundle $TM$ of a manifold $M$

⇨ Example: Parallel transport on a Riemannian manifold … ⇨ $\omega$: Levi-Civita connection

… integrating $\omega$ along a closed loop…




$$\theta = \int_{loop} \omega = \int_{area} d\omega$$




A cyclic change of shape

⇩

A net displacement in $G$

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The locomotion model is generally a dynamic model which can degenerate into kinematics, when we have a linear relation :

$$\eta + \mathcal{A}(r)\dot{r} = 0$$

- Defines a connexion on $G \times S$ [Ehresmann,1950]
- Encodes the model of all reaction forces

In bio-inspired robotics there are two well known cases where locomotion is modelled by a connexion…

First case: conservation law (ex. falling cat…)

$$\sigma = \sigma_{ref} + \sigma_{sh} = R(J\Omega + \alpha \dot{r}) = 0$$

$$\Omega = -(J^{-1}\alpha)(r)\dot{r}$$

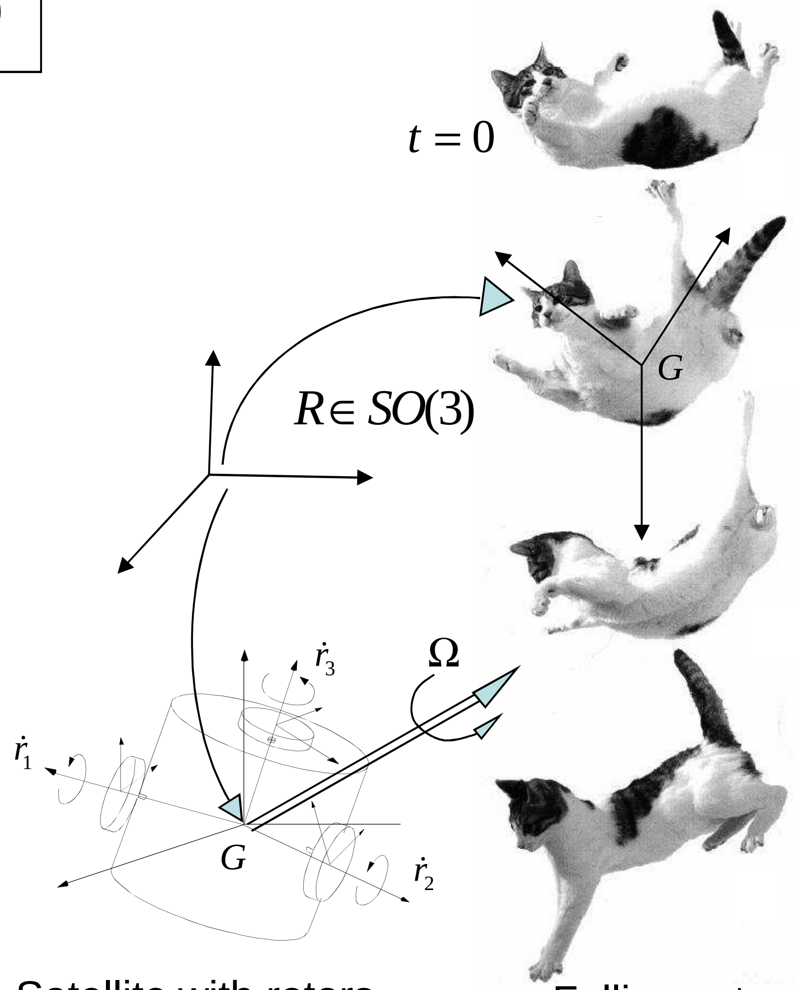
$$\mathcal{A}(r) = J^{-1}(r)\alpha(r)$$

Mechanical connexion

[Marsden 78, Montgomery 93]

Remark: Applying the same idea to translations of the reference frame…

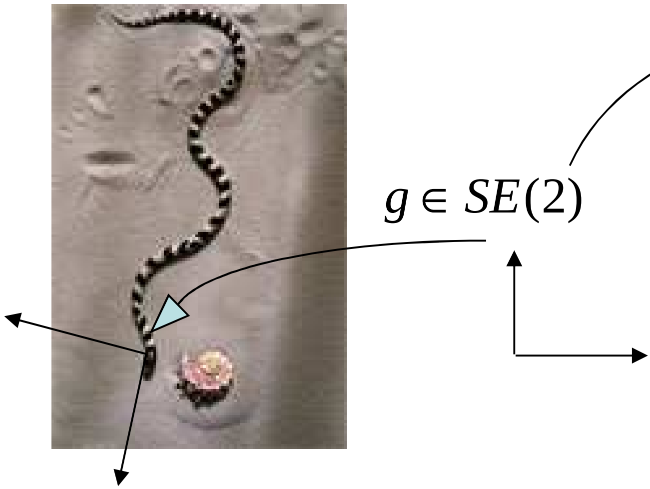
$$\Rightarrow \quad \mathcal{A}(r) = 0$$



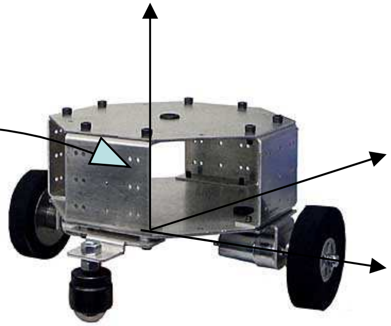

Satellite with rotors

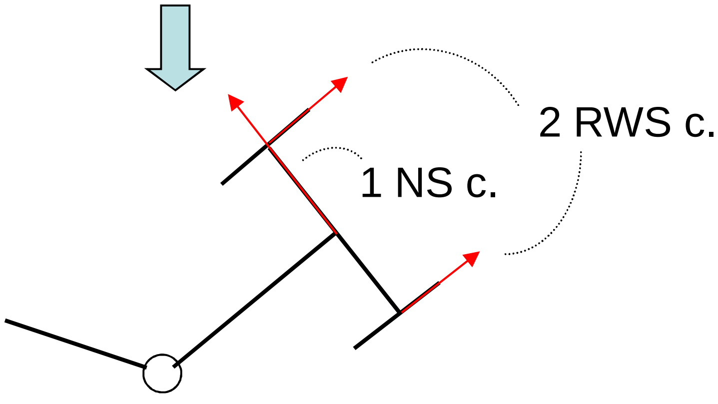
Falling cat

# Second case: snakes in lateral undulation



$$g \in SE(2)$$



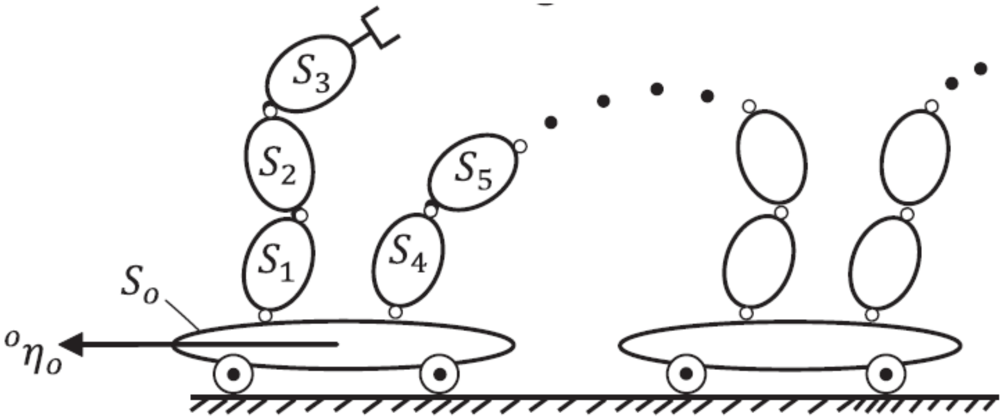



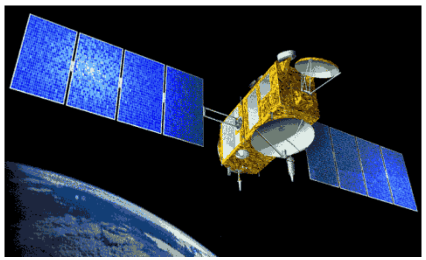

$$\eta = -\mathcal{A}(r)\dot{r}$$

$\mathcal{A}(r)$ : Principal kinematic connexion [Kelly & Murray, 95]

• First case where $F_{ext}$ requires no physics : when the contacts are ideal i.e. defined by kinematic constraints …

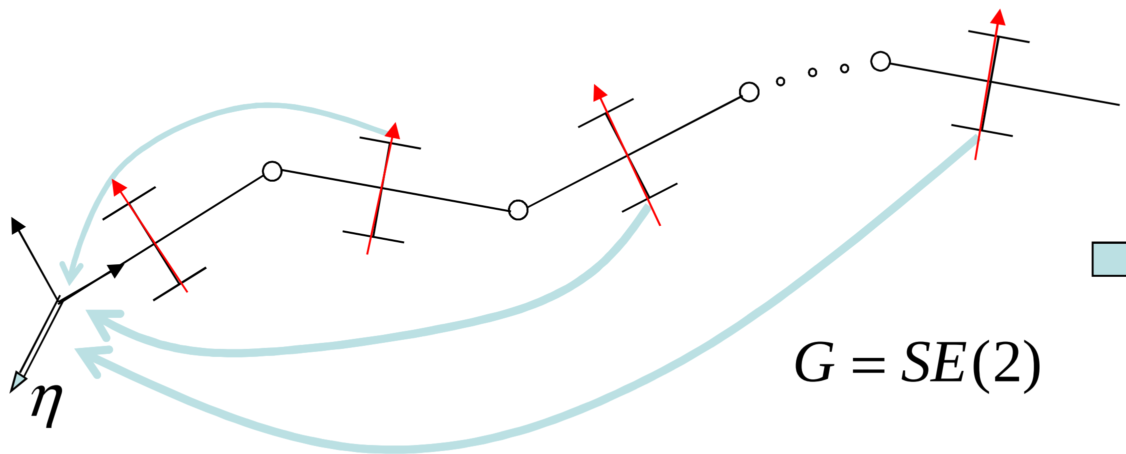
⇨ This is the case of (wheeled) Mobile Multi-body Systems (MMS) [Boyer&Ali, 11] :




 ,  ,  ,  ,  …

*"Hirose ACM-R5"*

This MMS is a serial assembly of passive wheeled axels connected by active joints…




*"ACMechanism"*

Reproduces the principle of Snake creeping:

Body undulations

⇩

Lateral reaction forces

⇩

Axial resultant

Gathering all the constraint equations on $G \times S$ :

⇨ $$A(r)\eta + B(r)\dot{r} = 0 \quad (*)$$

" $A(r)\eta + B(r)\dot{r} = 0$ " plays a crucial role in wheeled MMS classification…

⇨ 2 cases depending upon $rank(A)/3$

- Case 1 (fully constrained): $rank(A) = 3$ ⇨ Block partition of constraints…

ex: selection of 3 axels $(i, j, k)$ ←

$$\begin{pmatrix} \bar{A}(r) \\ \tilde{A}(r) \end{pmatrix} \eta + \begin{pmatrix} \bar{B}(r) \\ \tilde{B}(r) \end{pmatrix} \dot{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{matrix} \rightarrow (1) \\ \rightarrow (2) \end{matrix}$$

(1) ⇨ $\eta = -\bar{A}(r)^{-1}\bar{B}(r)\dot{r} = -\mathcal{A}(r)\dot{r}$

Principal kinematic connection: encodes the constraints of the wheels [Ostrowsky & Burdick,1999]

(2) ⇨ compatibility conditions

⇨ Allows to compute the other joint motions which preserve mobility…

- Case 2 (under constrained): $rank(A) < 3$

⇨ The MMS has not enough constraints to be governed by kinematics only!

Generalized Inversion of (*) ⇨ $$\eta = \underbrace{H(r)\,\eta_r}_{\in Ker(A)} + \underbrace{J(r)}_{= -A^{\dagger}B}\,\dot{r}$$

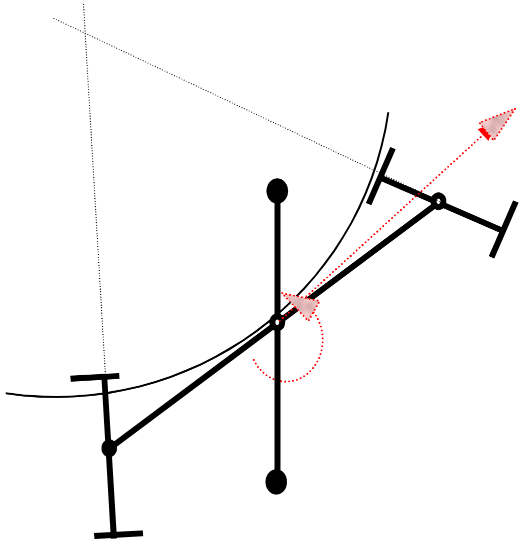
Where: $\eta_r$ kinematically undetermined!

⇨ To determine $\eta_r$ ⇨ we need dynamics…

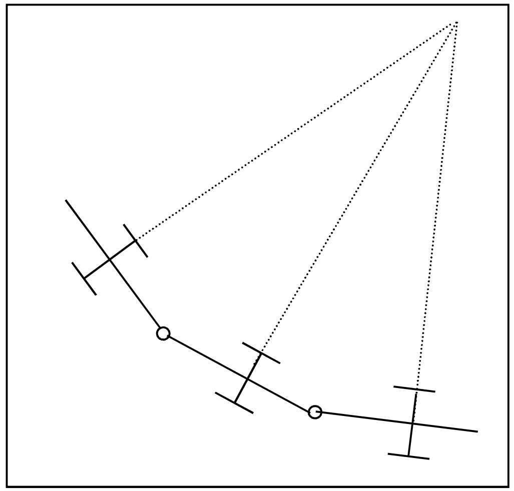
Remark : $Ker(A)$ = space of net velocities with $r$ locked!

⇨ Dynamics are required if the system can move with its shape locked…

Example: the snake board



Other example: Singular configurations of ACM… ⇨

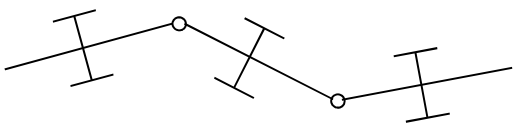
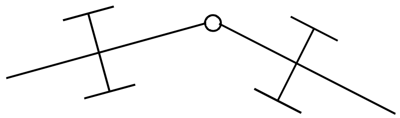
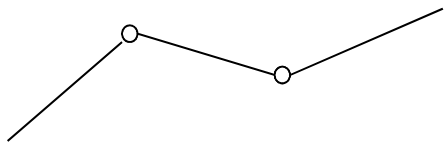


Projection of the unconstrained dynamics in $Ker(A)$ :

$$\Rightarrow \begin{pmatrix} \dot{\eta}_r \\ \dot{g} \end{pmatrix} = \begin{pmatrix} \mathcal{M}_r^{-1} F_r \\ g(H\eta_r + J\dot{r}) \end{pmatrix} \quad \text{, with: } \begin{cases} \mathcal{M}_r = H^T \mathcal{M} H \\ F_r = H^T(F - \mathcal{M}(\dot{H}\eta_r + \dot{J}\dot{r} + J\ddot{r})) \end{cases}$$

$$F = \cancel{F_{ext}} + F_{inert}$$

Contains all the sub cases:

| Case1: Pure Kinematic case (Snake robot) | Case2: Mixed Kinematic and Dynamic case (Snakeboard) | Case3: Pure dynamic case (fish robot) |
|---|---|---|
| $H = 0,\ J = -\mathcal{A} \neq 0$ | $H \neq 0,\ (J \neq 0,\ J = 0)$ | $H = 1,\ J = 0$ |
|  |  |  |

In the general case, a dynamic model is required…

To get it, take the Lagrangian: $l(g,r,\eta,\dot{r}) = \frac{1}{2}(\eta^T, \dot{r}^T)\begin{pmatrix} \mathcal{M} & m \\ m^T & M \end{pmatrix}\begin{pmatrix} \eta \\ \dot{r} \end{pmatrix} - U(g,r)$

⇨ Poincaré equations [Poincaré, 1901]:

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{\partial l}{\partial \eta}\right) - ad_{\eta}^{*}\left(\frac{\partial l}{\partial \eta}\right) = F_{ext}$$

⇨ Locomotion dynamics in state space :

$$\Rightarrow \quad \begin{pmatrix} \dot{\eta} \\ \dot{g} \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} F \\ g\eta \end{pmatrix}$$

$\mathcal{M}$ → locked inertia tensor

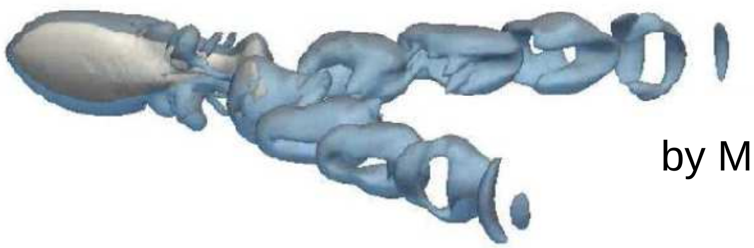
$F$ → $= F_{ext} + F_{inert}$ : locked forces

$\dot{g} = g\eta$ → reconstruction eq. from $\eta$ to $g$

In general, $F_{ext}$ requires to solve the contact dynamics of the system / world …

Which can be extremly difficult…

For example in swimming, $F_{ext}$ requires to solve Navier-Stokes equations!



by M. Bergmann

However, there exists some simple situations where $F_{ext}$ only requires geometry (no physics)… two cases…

• Snd case: when $F_{ext}$ is Lagrangian [Birkhoff, 66], i.e. when there exists $l_{ext}(r,\eta,\dot{r})$ …

s.t.: $F_{ext} = -\frac{d}{dt}\left(\frac{\partial l_{ext}}{\partial \eta}\right) + ad^*_\eta\left(\frac{\partial l_{ext}}{\partial \eta}\right)$ ⇒ Locomotion dynamics:

$$\frac{d}{dt}\left(\frac{\partial (l+l_{ext})}{\partial \eta}\right) - ad^*_\eta\left(\frac{\partial (l+l_{ext})}{\partial \eta}\right) = 0$$

But then…

if at $t=0$ , $\frac{\partial (l+l_{ext})}{\partial \eta} = 0$ ⇒ $\frac{\partial (l+l_{ext})}{\partial \eta} = 0 \ , \ \forall t$ : Locomotion dynamics:

Swimming at high Reynolds in a quiescent potential flow:

Tensor of virtual inertia = solid+ added

$$l_{ext} = T_{fluid} \Rightarrow l + l_{ext} = \frac{1}{2}(\eta^T, \dot{r}^T)\begin{pmatrix} \tilde{\mathcal{M}} & \tilde{m} \\ \tilde{m}^T & \tilde{M} \end{pmatrix}\begin{pmatrix} \eta \\ \dot{r} \end{pmatrix}$$

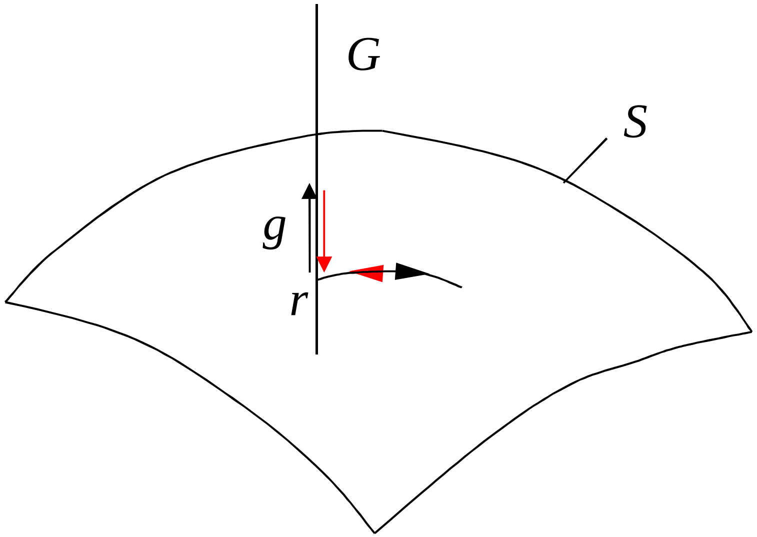
⇒ Conservation law of kinetic momentum: $\tilde{\mathcal{M}}\eta + \tilde{m}\dot{r} = 0$ ⇒ $\eta + \tilde{\mathcal{A}}(r)\dot{r} = 0$

Mechanical connection: encodes kinetic exchanges body / fluid…[Kanso, 2005, 09…]

Remarks:

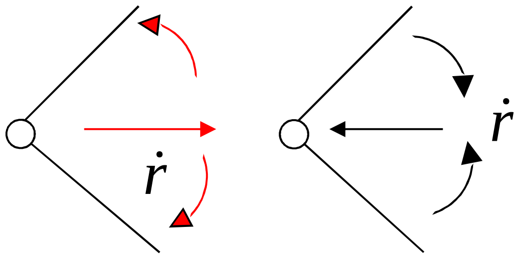
1°) Can change the orientation but also the position

2°) A cyclic change of one dof shape ⇒ no phase shift on $G$




⇔

Theorem of Scallop …




3°) similar context at low Reynolds with viscous forces…

At high Reynolds, kinetic conservation laws can be prolonged to the case with vorticity… applying the balance of impulses wrench [Saffmann, 92] gives:

$$\left\{ \begin{pmatrix} p_{sh} \\ \sigma_{sh} \end{pmatrix} + \begin{pmatrix} p_{rf} \\ \sigma_{rf} \end{pmatrix} + \underbrace{\begin{pmatrix} \int_{\partial B} x \times (n \times u_\omega) da + \frac{1}{2} \int_F x \times \omega \, dv \\ -\frac{1}{2} \int_{\partial B} \|x\|^2 (n \times u_\omega) da - \frac{1}{2} \int_F \|x\|^2 \omega \, dv \end{pmatrix}}_{\text{Impulse wrench due to vorticity}} = \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{\text{Starting at rest}} \right.$$

Impulse wrench due to vorticity

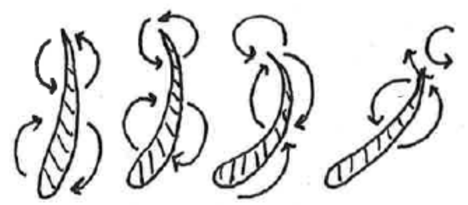
Starting at rest

Animals generate (and control) vortices to move efficiently!

A scallop escapes from its theorem…

Generate vorticity for what?

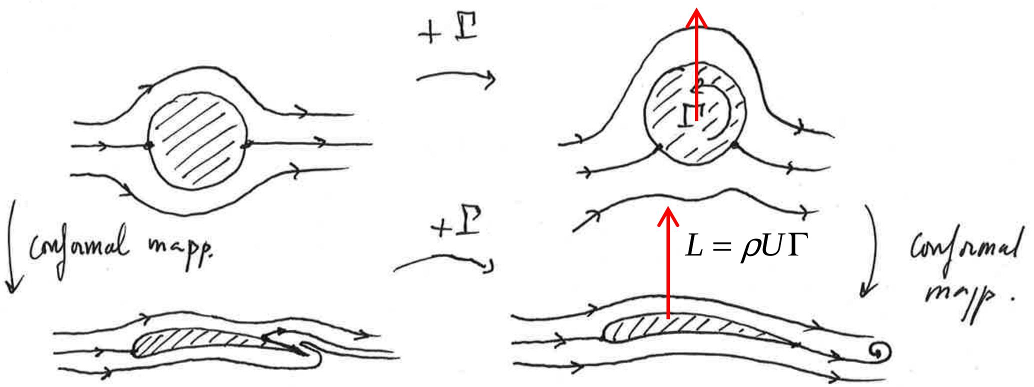
- To manoeuver (turn, plung…) in the case of fishes



More generally…

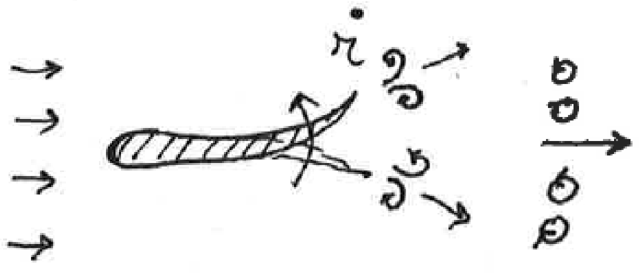
- To generate lift (used for sustentation, thrust…) against drag:

Remind the basic picture of lift generation in steady aerodynamics…

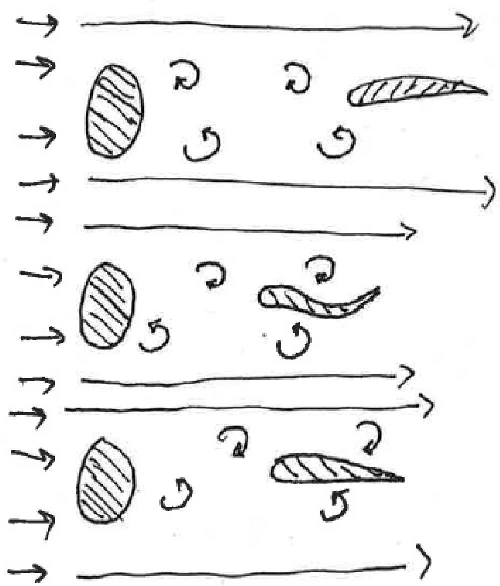



Shed vorticity ⇩ bound vorticity ⇩ circulation ⇩ lift!

- Wake energy recovering:



- Flow energy recovering :



vidéo

[JFM (subm.) 11]

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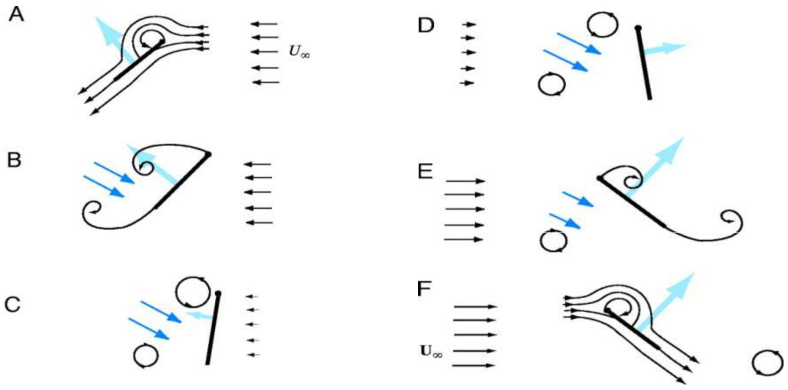
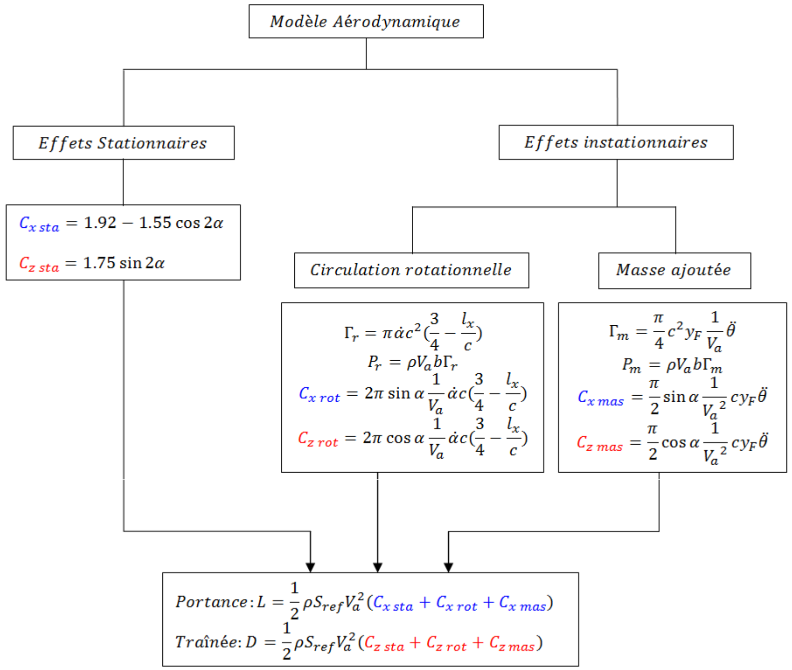
## Exemple of the flapping wing

Understand the mechanisms [vidéo](#) [vidéo](#)



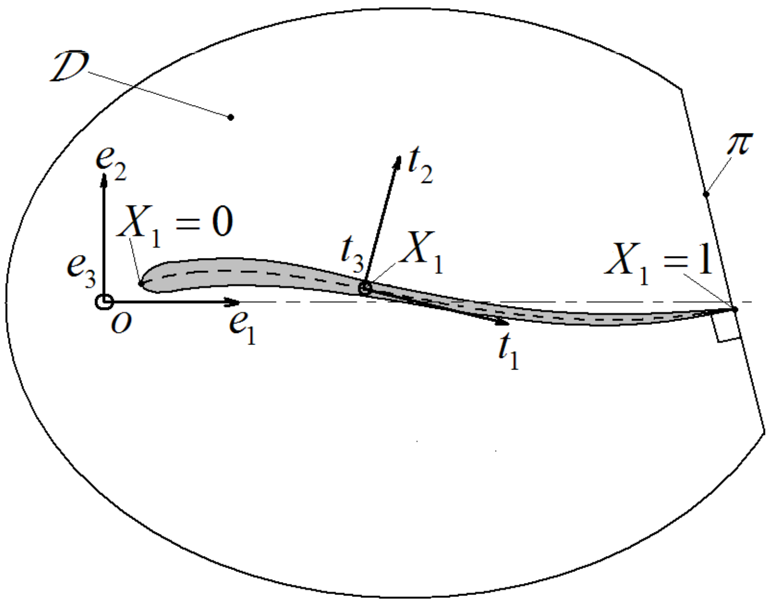

Put them in the Poincaré equations → Modelling the contact forces

$$\begin{pmatrix} \mathcal{M} & m_a & m_e \\ m_a^T & M_{aa} & M_{ae} \\ m_e^T & M_{ea} & M_{ee} \end{pmatrix} . \begin{pmatrix} \eta \\ \ddot{r}_a \\ \ddot{r}_e \end{pmatrix} + \begin{pmatrix} F_{in} \\ Q_{a,in} \\ Q_{e,in} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ K_{ee}(r_e)r_e \end{pmatrix} + \begin{pmatrix} F_f \\ Q_{fa} \\ Q_{fe} \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_a \\ 0 \end{pmatrix}$$

Exemple of the anguilliform swimming [TRO 08, JNLS 10, JFM 11]

L.A.E.B.T. [Lighthill, 1970] ⇨ **Mechanism of kinetic energy amplification** ⇨

1°) Control-volume $\mathcal{D}$ = ∞te radius hemisphere boun ded by caudal plane $\pi$




**2°) a kinetic balance of the fluid in $\mathcal{D}$**

$$Te_1 + Le_2 = -\frac{\partial}{\partial t}\int_0^1 mV_2 t_2 dX_1 + \left[ mV_2 V_1 t_2 - \frac{1}{2} mV_2^2 t_1 \right]_{X_1=1}$$

- $Te_1 + Le_2$ → Thrust
- $mV_2 V_1 t_2$ → Shed impulse
- $\frac{1}{2} mV_2^2 t_1$ → Shed kinetic energy

**Self propell the fish body** ⇦ Shed kinetic energy

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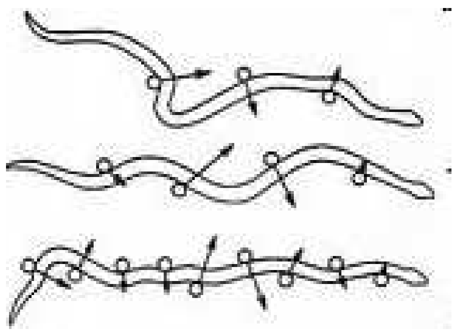
⇨ Exemple of the Snake creeping [TRO 11, TRO 12]



- External dynamics…

When the total number of independent constraints $\geq 6$ ⇨

⇨ Locomotion entirely ruled by kinematics of contact and controlled strains

$$\eta_o = f(g_o, \dot{\xi}_d, \xi_d, \xi'_d, \xi''_d, \ldots)$$ ⇔ Forward locomotion kinematics

- Internal dynamics…

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\dfrac{\partial \mathcal{L}}{\partial \eta}\right) - ad_\eta^T\left(\dfrac{\partial \mathcal{L}}{\partial \eta}\right) + \dfrac{\partial}{\partial X}\left(\dfrac{\partial \mathcal{L}}{\partial \xi}\right) - ad_\xi^T\left(\dfrac{\partial \mathcal{L}}{\partial \xi}\right) = \bar{F} \\ \left(\dfrac{\partial \mathcal{L}}{\partial \xi}\right)_\pm = \pm \bar{F}_\pm \end{cases}$$

$\rightarrow$ Poincaré equations of a Cosserat-beam

With: $\mathcal{L} = \mathcal{T} - \mathcal{U} = \dfrac{1}{2}\eta^T \mathcal{M} \eta - \Lambda^T(\xi - \xi_d(t))$

vidéo

# Questions …?