Nonlinear Control of PVTOL Vehicles subjected to Drag and Lift Forces

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Outline

Aerial vehicles

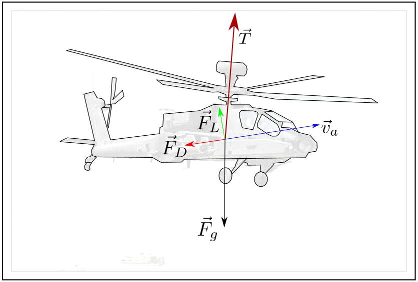
Rotary-wings in hovering phase

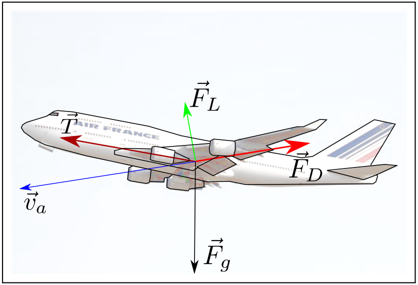
- Lift forces are negligible
- Thrust counteracts the weight
- High energy consumption

Fixed-wings in cruising phase

- Lift compensates the weight
- Thrust counteracts the drag
- Low energy consumption

Aerial vehicles





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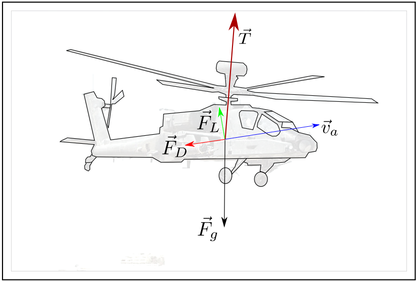
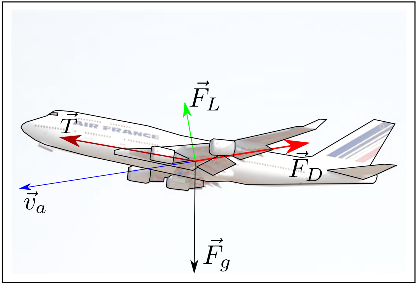




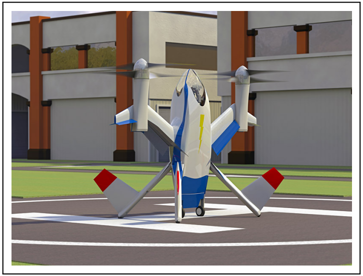
Rotary-wings in hovering phase

- Nonlinear feedback methods developed by neglecting the aerodynamic forces

Fixed-wings in cruising phase

- Feedback methods for linearized aerodynamic models
- Only local stability is concerned

Convertible vehicles



In vertical take-off phase

- the aerodynamic forces are negligible
- control design similar to the one for hovering phase

In cruising flight phase

- the aerodynamic forces are preponderant
- control design similar to the one for cruising phase

Control requirement

Robust transition maneuvers from take-off to cruising flight

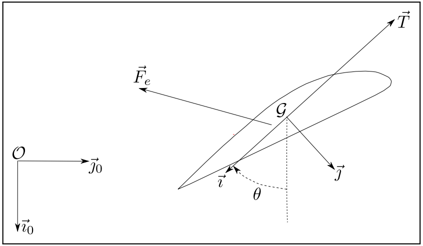
Objectives

- Analysis and modeling of the aerodynamic reaction forces
- Development of nonlinear controllers by taking into account the nonlinearities of the aerodynamic reaction forces
- Development of a unique robust control law from take-off to cruising flight

Problem statement, notation and system modeling

A single actuated vehicle immersed in air which exerts reaction forces is considered in the 3-D Lie group $SE(2)$

- θ angle between $\vec{\imath}_0$ and $\vec{\imath}$
- $\dot{x}$ linear velocity of G



System's dynamics

$$
\begin{aligned}
m\ddot{x} &= -TR(\theta)e_1 + mge_1 + F_a(\dot{x}_a, \theta) \\
J\dot{\omega} &= \mathcal{M} + \mathcal{M}_a \\
\dot{x}_a &= \dot{x} - \dot{x}_w
\end{aligned}
$$

Assumption

Complete torque actuation $\Longrightarrow$ ω as control input

Aerodynamic reaction forces

- Interactions between a solid body and the surrounding fluid are governed by *Navier–Stokes equations*
- They are a set of nonlinear partial differential equations involving
  - *viscosity,*
  - *compressibility,*
  - and *density* of the fluid
- Solving the equations requires spatial integration over the shape of the body which typically does not yield closed-form expressions

Analytical expression of the aerodynamic forces

Lift and drag decomposition

$$F_a = F_L + F_D$$
$$F_L = k_a |\dot{x}_a| c_L(\cdot) \dot{x}_a^{\perp}$$
$$F_D = -k_a |\dot{x}_a| c_D(\cdot) \dot{x}_a$$

Buckingham π theorem

$$c_L = c_L(R_e, M, \alpha)$$
$$c_D = c_D(R_e, M, \alpha)$$

- $k_a := \frac{1}{2}\rho\Sigma$, ρ is the air density, Σ is the length of the body
- $c_L(\cdot)$ is the *lift coefficient*
- $c_D(\cdot) > 0$ is the *drag coefficient*
- R_e is the *Reynolds number*
- M is the *Mach number*
- α is the *angle of attack*

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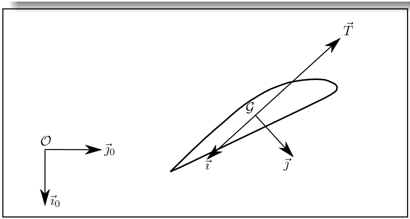

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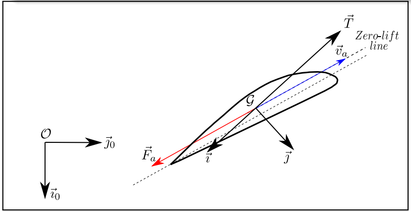

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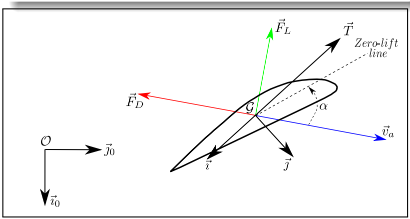

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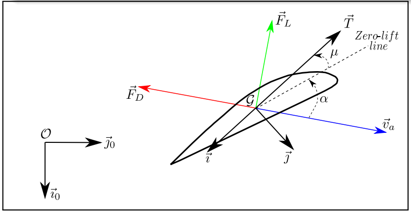

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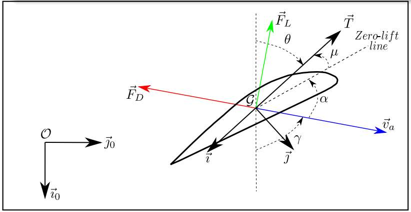
The angle of attack

$$\alpha(\dot{x}_a, \theta) = \pi + \theta - \gamma(\dot{x}_a) - \mu$$



Flow regimes: a thumb criterion

- *Subsonic flow* if $M < 0.8$.
- *Transonic flow* if $0.8 < M < 1.2$.
- *Supersonic flow* if $1.2 < M < 5$.
- *Hypersonic flow* if $M > 5$.

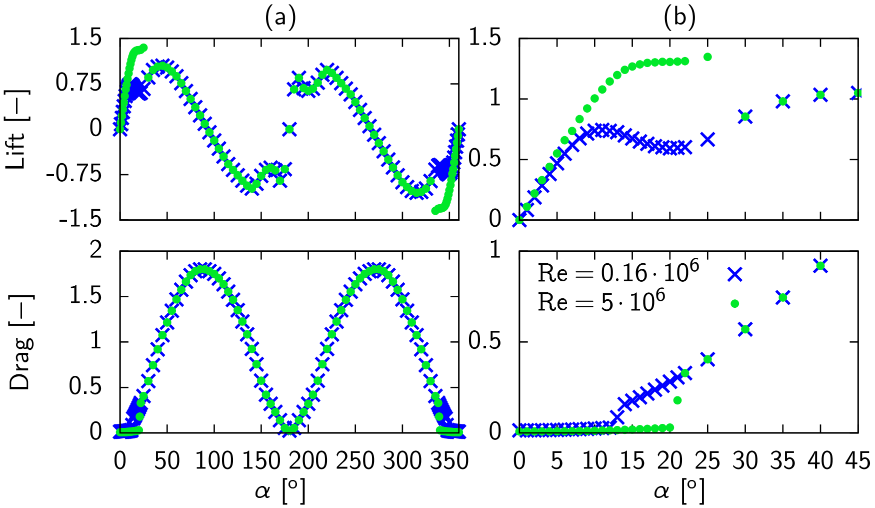
Aerodynamic characteristics in the low-subsonic regime

Mach number $M < 0.5$

- The air ρ is approximately constant
- M does not greatly affect c_L and $c_D \Rightarrow$
- $c_L = c_L(R_e, \alpha)$, $c_D = c_D(R_e, \alpha)$
- Typical data for c_L and c_D

Aerodynamic characteristics in the low-subsonic regime

Lift and drag coefficient for NACA airfoil 0021



Single frequency approximation in low-subsonic regime

Assumption: bi-symmetric body

$$\begin{cases} c_D(\alpha) = c_D(-\alpha) \\ c_L(\alpha) = -c_L(-\alpha) \end{cases}$$

$$\begin{cases} c_D(\alpha) = c_D(\alpha+\pi) \\ c_L(\alpha) = c_L(\alpha+\pi) \end{cases}$$

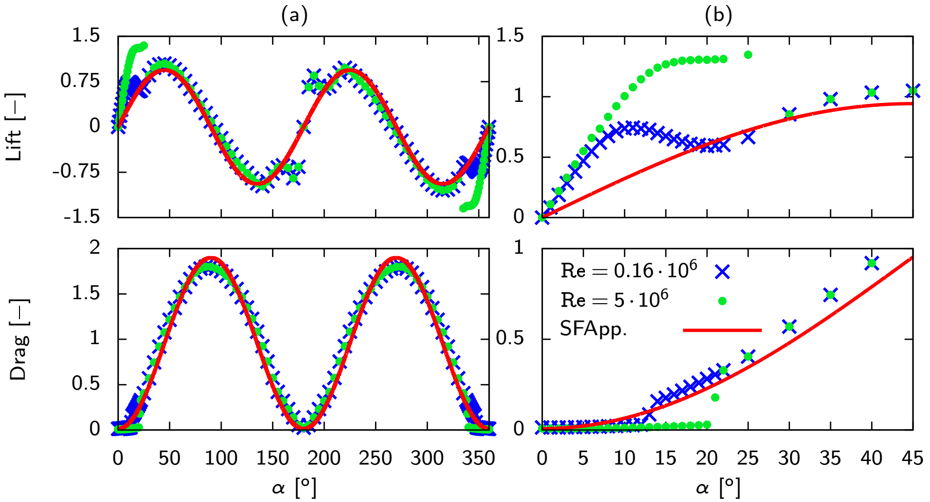
Single frequency approximation

$$\begin{cases} c_D(\alpha) = c_{D_0} + 2c_1 \sin^2(\alpha) \\ c_L(\alpha) = c_1 \sin(2\alpha) \end{cases}$$

Important fact:

the value for c_1 must be properly chosen, i.e. solution to least square minimization problem.

Single frequency approximation in low-subsonic regime



Introduction to the control problem

System's dynamics

$$m\ddot{x} = -TR(\theta)e_1 + mge_1 + F_a(\dot{x}_a,\theta)$$
$$\dot{\theta} = \omega$$

Spherical shapes (Hua, Hamel, Morin & Samson 2009)

$F_a(\dot{x}_a,\theta) = F_a(\dot{x}_a) \quad \Rightarrow \quad$ Control design much simplified

Example. Let $\dot{\tilde{x}} = \dot{x} - \dot{x}_r(t)$. Then $\dot{\tilde{x}} \equiv 0 \iff$

$$\boxed{-TR(\theta)e_1 + F(\dot{x},t) = 0}$$

$F(\dot{x},t)$: "apparent external forces"

The alike-spherical characteristics

Class of functions c_L and c_D for which $T \longrightarrow T_p$ yields

$$m\ddot{x} = -T_pR(\theta)e_1 + mge_1 + F_p(\dot{x}_a)$$

The case of single frequency approximation of c_L and c_D

Assume that the resultant of the aerodynamic forces is of the form

$$F_a = k_a|\dot{x}_a|\Big[c_L(\alpha)S - c_D(\alpha)I\Big]\dot{x}_a.$$

Main result

$$m\ddot{x} = -TR(\theta)e_1 + mge_1 + F_a(\dot{x}_a, \theta)$$

can be transformed into the form

$$m\ddot{x} = -T_pR(\theta)e_1 + mge_1 + F_p(\dot{x}_a)$$

with F_p independent of θ, if

$$\begin{cases} c_D(\alpha) = c_{D_0} + 2c_1\sin^2(\alpha) \\ c_L(\alpha) = c_1\sin(2\alpha) \end{cases}$$

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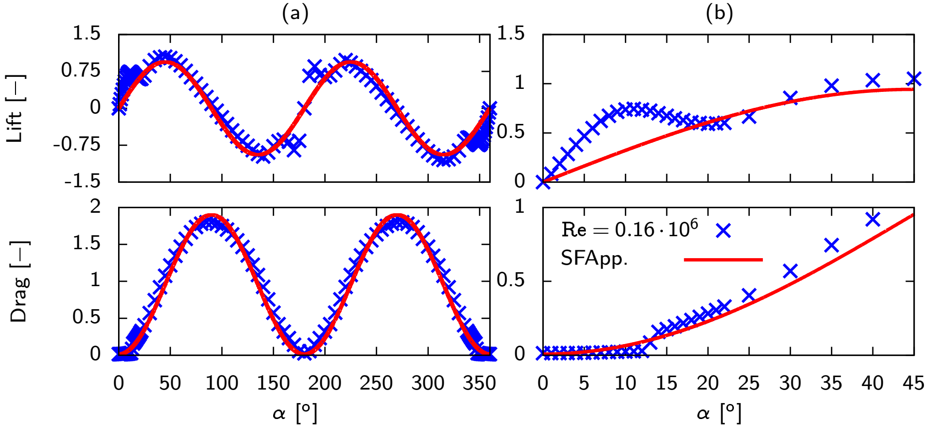
Control design

Once the transformation is done, the control design is similar to the one for systems subjected to drag forces only

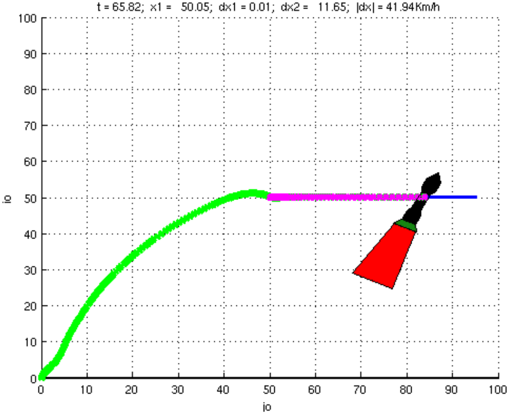
A control solution based on (Hua et al. 2009)

- Control laws guarantee (almost)-global stability domain for $x_r(t)$
- Ensures robustness properties

Simulation from hovering to cruising flight:



From hovering to cruising flight



Conclusions

Absence of the stall phenomena

- Control problem recasted to the one of a spherical shape
- Control design is similar to the one for systems subjected to drag
- Using (Hua et al. 2009) (almost)-global stability and robustness to constant disturbances can be achieved

Stall phenomena

- Existence of equilibrium point is ensured
- Uniqueness of the equilibrium point is not ensured
- Reference trajectories ensuring continuity of $\theta_e(t)$
- Global stability difficult to achieve
- Laws ensuring local stability of x_r or $\dot{x}_r$ have been developed
- Robustness to unmodelled dynamics are achieved

Future work

- Nature of the equilibrium points
- Positivity of the thrust
- Convergence to the e.p. endowed with the smallest drag
- Enlarge the domain of attraction
- Take into account the $\omega-$dynamics
- 3D-Case ...

Bibliography

Hua, M. D., Hamel, T., Morin, P. & Samson, C. (2009). A control approach for thrust-propelled underactuated vehicles and its application to vtol drones, *0* (8): –.