Robust Rendezvous Control of UAVs with Collision Avoidance and Connectivity Maintenance

Esteban Restrepo[1,2], Antonio Loría[2,3], Ioannis Sarras[1], Julien Marzat[1]

[1]DTIS, ONERA, Université Paris-Saclay, F-91123 Palaiseau, France
[2]L2S-CentraleSupélec, Université Paris-Saclay, F-91192 Gif-sur-Yvette, France
[3]CNRS, F-91192 Gif-sur-Yvette, France

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Summary

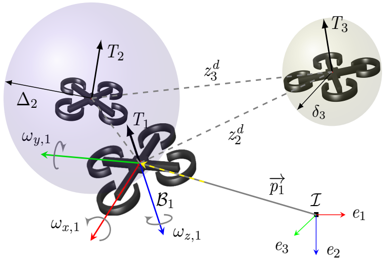
Plan

Safety-aware deployment

Rendezvous in formation in realistic settings

Constraints:

- Local knowledge
- Limited interaction range
- Minimal safety distance
- Underactuated nonlinear dynamics
- Disturbances: aerodynamic, modeling uncertainties, *etc.*



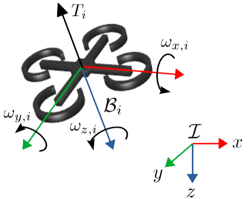
Objective

Find **distributed** controllers that achieve a desired static formation for the multi-drone system with guaranteed **connectivity** maintenance and **collision avoidance** in the presence of **disturbances**.

System modeling

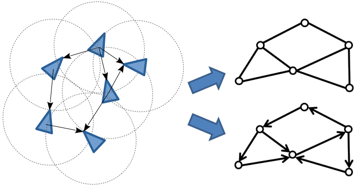
Single agent: "mixed" model

$$\dot{p}_i = v_i$$
$$\dot{v}_i = -\frac{T_i}{m_i}\mathfrak{R}_i e_3 + g e_3 + \theta_i(t)$$
$$\dot{\mathfrak{R}}_i = \mathfrak{R}_i S(\omega_i)$$



p_i: Cartesian position
v_i: velocity in $\mathcal{I}$
T_i: thrust
$\mathfrak{R}_i$: rotation matrix
ω_i: angular rate in $\mathcal{B}_i$
g: gravity acceleration
m_i: mass

Multiple agents: Each agent communicates only with its neighbors



Graphs

The information exchange is defined by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

- Nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$
- Edges $\mathcal{E} \subset \mathcal{V}^2$, $|\mathcal{E}| = M$

Problem formulation

Rendezvous problem

Reach a desired static formation

$$\lim_{t\to\infty} p_i(t) - p_j(t) = z_k^d$$

$$\lim_{t\to\infty} v_i(t) = 0$$

Inter-agent constraints

$$\delta_k < |p_i(t) - p_j(t)| < \Delta_k$$

$\forall$ i and j such that

$$\delta_k < |p_i(0) - p_j(0)| < \Delta_k$$

Notation: for each edge $e_k = (i, j) \in \mathcal{E}$

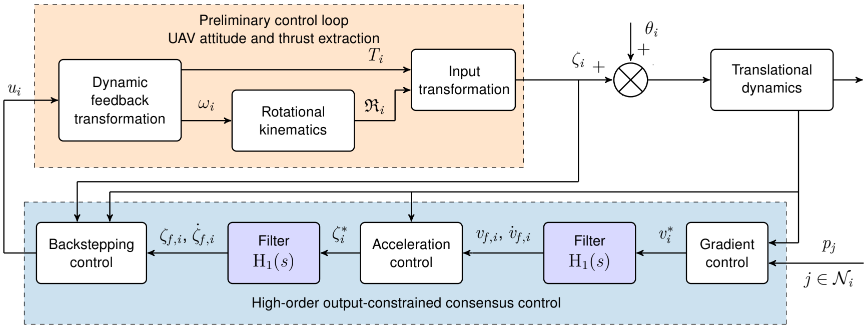
- z_k^d: desired relative position in formation
- Δ_k: maximum distance to guarantee connectivity
- δ_k: minimum distance to guarantee collision avoidance

Plan

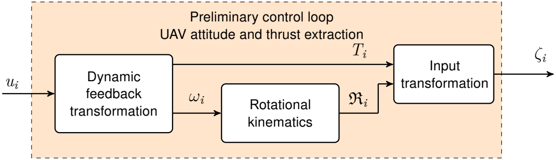
Control architecture

Hierarchical control

1. Faster attitude kinematics → thrust and attitude extraction via dynamic feedback
2. Consensus control for high-order chains of integrators



Preliminary control loop (I)



- The translational subsystem may be modeled by as a **double integrator**

$$\dot{p}_i = v_i$$
$$\dot{v}_i = \zeta_i + \theta_i(t)$$

$$\zeta_i := -\frac{T_i}{m_i}\mathfrak{R}_i e_3 + g e_3$$

Solve for T_i and $\mathfrak{R}_i$!

Dynamic input transformation[1]

Design the inputs ω_i and an update law for T_i such that

$$\dot{\zeta}_i = u_i$$

[1] Dongjun Lee (2012). "Distributed backstepping control of multiple thrust-propelled vehicles on a balanced graph". In: Automatica 48.11, pp. 2971–2977

Preliminary control loop (II)

$$\omega_i = \left[\frac{m_i \tilde{\nu}_{i,y}}{T_i}, \; -\frac{m_i \tilde{\nu}_{i,x}}{T_i}, \; \omega_{zi} \right]^\top, \qquad \dot{T}_i = -c_3 \, T_i - m_i \tilde{\nu}_{i,z}.$$

where

$$\tilde{\nu}_i := \mathfrak{R}_i^\top \nu_i, \quad \nu_i := u_i - \frac{c_3}{m_i} T_i \mathfrak{R}_i e_3, \quad c_3 > 0$$

- The underactuated system is transformed into a **third-order integrator**

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \zeta_i + \theta_i(t) \\ \dot{\zeta}_i &= u_i \end{aligned}$$

Rmk. The transformation is only valid for $T_i \neq 0$.

$$\zeta_i = -\frac{T_i}{m_i} \mathfrak{R}_i e_3 + g e_3 \qquad \Longrightarrow \qquad |g e_3 - \zeta_i| \neq 0$$

This condition can be satisfied if we guarantee via the controller design that

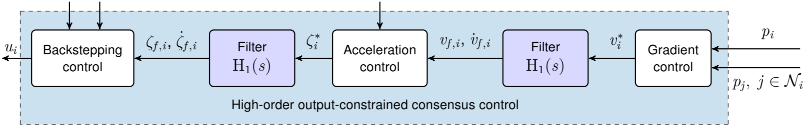
$$|\zeta_i| \leq \bar{\zeta}_M < g$$

Control methodology

$$\dot{p} = v$$
$$\dot{v} = \zeta + \theta(t)$$
$$\dot{\zeta} = u$$

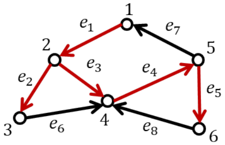
Consensus of high-order systems under output-constraints[2]

- Consensus under inter-agent constraints
- Lyapunov design – backstepping



[2] Esteban Restrepo et al. (2021). "Robust Consensus of High-Order Systems under Output Constraints: Application to Rendezvous of Underactuated UAVs". In: IEEE Transactions on Automatic Control. Submitted as a regular paper on April 2021.

Edge transformation[3]



Incidence matrix $E \in \mathbb{R}^{N \times M}$

$$[E]_{ik} := \begin{cases} +1, & \text{if } i \text{ is the initial node of edge } e_k \\ -1, & \text{if } i \text{ is the terminal node of edge } e_k \\ 0, & \text{otherwise} \end{cases}$$

Edge transformation:

$$z_k = p_i - p_j, \quad k \leq M, \quad (i,j) \in \mathcal{E}$$
$$\tilde{z}_k = z_k - z_k^d$$

Compact form:

$$z = \left[E^\top \otimes I_3\right] p$$
$$\tilde{z} = \left[E^\top \otimes I_3\right] p - z^d$$

Consensus $(p_1 = \cdots = p_N) \iff$ Stabilization of the origin $(z = 0)$

- Position subsystem in the edge coordinates

$$\dot{\tilde{z}} = \left[E^\top \otimes I_3\right] v$$

[3] D. Zelazo et al. (2007). "Agreement via the edge Laplacian". en. In: 46th IEEE Conference on Decision and Control. New Orleans, LA, USA, pp. 2309–2314

Barrier Lyapunov function

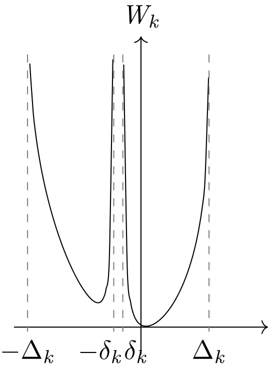
Constraints set

$$\tilde{z}_k \in \mathcal{D}_k := \{\tilde{z}_k \in \mathbb{R}^n \, : \, \delta_k < |\tilde{z}_k + z_k^d| < \Delta_k\}$$

Barrier Lyapunov function: $W_k : \mathcal{D}_k \to \mathbb{R}_{\geq 0}$

$$W_k(\tilde{z}_k) := \frac{1}{2}\left[|\tilde{z}_k|^2 + B_k(\tilde{z}_k + z_k^d)\right]$$

- Non-negative function B_k[4,5]
- $B_k(z_k^d) = 0$ and $\nabla B_k(z_k^d) := \frac{\partial B_k}{\partial z_k}\big|_{\tilde{z}_k = z_k^d} = 0$
- $B_k(\tilde{z}_k + z_k^d) \to \infty$ as $|\tilde{z}_k + z_k^d| \to \partial\mathcal{D}$



[4] A. G. Wills et al. (2002). "A recentred barrier for constrained receding horizon control". In: Proceedings of the 2002 American Control Conference (IEEE Cat. No.CH37301). Vol. 5, 4177–4182 vol.5

[5] Keng Peng Tee et al. (2012). "Control of state-constrained nonlinear systems using integral barrier Lyapunov functionals". In: 2012 IEEE 51st IEEE Conference on Decision and Control (CDC). IEEE, pp. 3239–3244

Consensus under constraints[6,7]

Constrained consensus control

$$v_i^* = -\, c \sum_{k \leq M} [E]_{ik} \, \nabla W_k(\tilde{z}_k)$$

$$v^* = -\, c\, E \nabla W(\tilde{z}), \quad W(\tilde{z}) = \sum_{k \leq M} W_k(\tilde{z}_k)$$

Closed loop

$$\dot{\tilde{z}} = -\, c\, L_e \nabla W(\tilde{z})$$

$$L_e := \left[E^\top E \otimes I_3\right]$$

Proof of stability

$$\dot{W}(\tilde{z}) = -c \nabla W(\tilde{z})^\top L_e \nabla W(\tilde{z}) < 0$$

- Asymptotic stability at the origin
- Respect of the constraints

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[6] Esteban Restrepo et al. (2020b). "Stability and robustness of edge-agreement-based consensus protocols for undirected proximity graphs". In: International Journal of Control, pp. 1–9

[7] Esteban Restrepo et al. (2020a). "Edge-based Strict Lyapunov Functions for Consensus with Connectivity Preservation over Directed Graphs". In: Automatica. To appear.

Command filtered backstepping[8]

$$\dot{\tilde{z}} = \left[E^\top \otimes I_3\right] v$$
$$\dot{v} = \zeta + \theta(t)$$
$$\dot{\zeta} = u$$

Virtual inputs

$$v^* = -\,c_1\left[E \otimes I_3\right]\nabla W(\tilde{z}), \qquad W(\tilde{z}) = \sum_{k \le M} W_k(\tilde{z}_k)$$

$$\zeta^* = \mathrm{sat}\left(-c_2\tilde{v} + \dot{v}^*\right), \qquad \tilde{v} = v - v^*$$

Needs high-order derivatives of $W(\tilde{z})$!

[8] J. A. Farrell et al. (2009). "Command Filtered Backstepping". In: IEEE Transactions on Automatic Control 54.6, pp. 1391–1395

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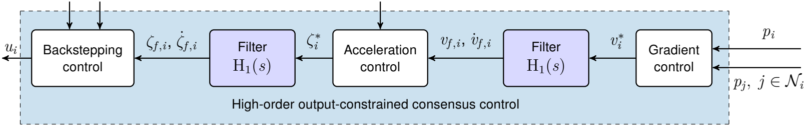


Control input

$$\tilde{v} := v - v_f, \quad \tilde{\zeta} = \zeta - \zeta_f$$
$$u = -\,c_3\tilde{\zeta} + \dot{\zeta}_f - \tilde{v}$$

[8] J. A. Farrell et al. (2009). "Command Filtered Backstepping". In: IEEE Transactions on Automatic Control 54.6, pp. 1391–1395

Controller review

- Almost-everywhere **practical input-to-state stability**[2]
- Convergence to consensus in the undisturbed case
- Inter-agent constraints: **connectivity maintenance** and **collision avoidance**
- Valid for **undirected** graphs, **directed spanning trees** and **directed cycles**
- **Bounded** thrust input
- Well suited for practical implementation

[2] Esteban Restrepo et al. (2021). "Robust Consensus of High-Order Systems under Output Constraints: Application to Rendezvous of Underactuated UAVs". In: IEEE Transactions on Automatic Control. Submitted as a regular paper on April 2021.

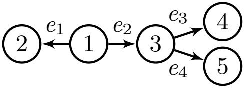
Plan

Experimental setup

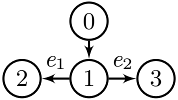
- 5 DJI Tello®
- Robot Operating System
- Optitrack motion capture system based on active IR cameras
- Rendezvous in formation
- Each agent has a limited range Δ_k and a safety distance δ_k

Cases:

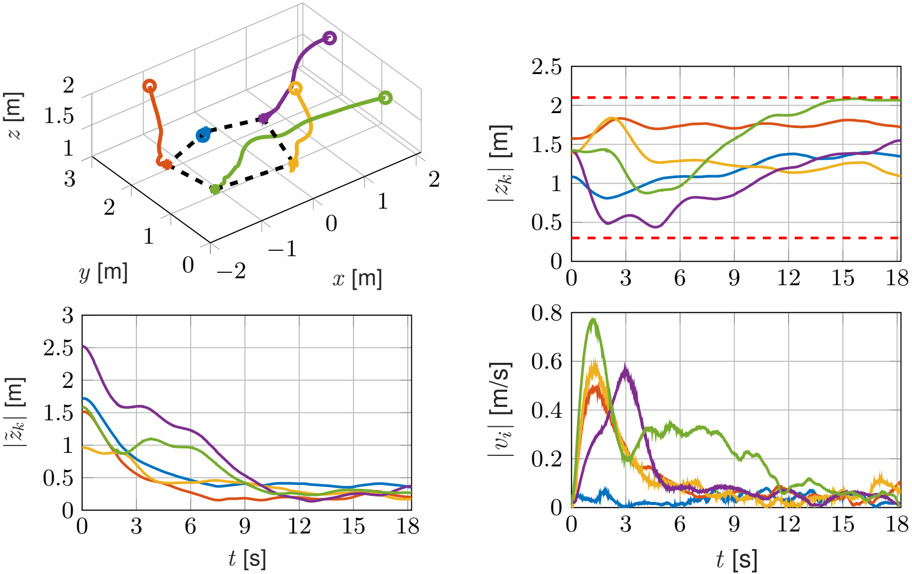
- Directed spanning tree



- Virtual leader



Directed spanning tree



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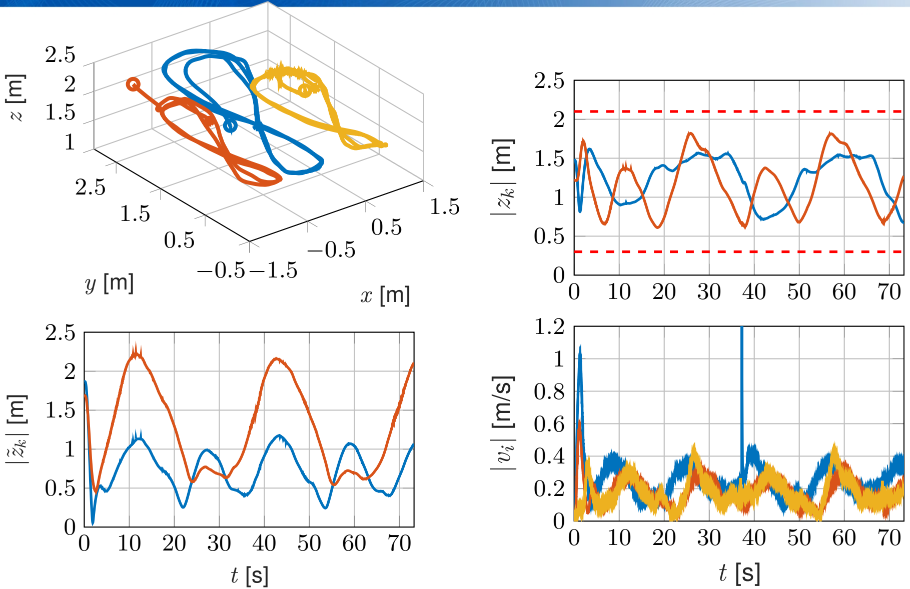

Plan

Conclusions

Contributions

- Edge-based rendezvous with connectivity maintenance and collision avoidance of underactuated UAVs
- Strong stability and robustness properties $\rightarrow$ practical ISS
- Undirected and directed topologies
- Experimental validation

Further research

- More general directed graph topologies
- Open networks
- Additional constraints: obstacle avoidance, quantization, *etc.*

Thank you for your attention !

Questions ?

contact: esteban.restrepo@onera.fr