On the guidance of a UAV under unknown wind disturbances

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Outline

Problem definition

- Path-following guidance in 3D
- Non-singular kinematics
- Wind disturbances

Outline

Reference frames

- **Inertial reference frame** $\{i\}$: Origin at O_i with right-hand axes oriented in north, east and down (NED)
- **Body-fixed frame** $\{b\}$: Origin O_b at the vehicle's COM with right-hand axes aligned with the principal axes of inertia
- **Serret-Frenet frame** $\{f\}$: Origin at O_f and with axes defined by the tangent T, normal N and bi-normal B vectors of the desired spatial path

Non-singular Serret-Frenet frame

Evolution of f-frame will serve as new degree-of-freedom

Kinematics

- $\mathscr{X}_{bi}^{b} := \mathrm{col}(x_{bi}^{b}, y_{bi}^{b}, z_{bi}^{b})$: Position of $\{b\}$ wrt $\{i\}$ (subscript) when expressed in $\{b\}$(superscript)
- $\Theta_{ib} := \mathrm{col}(\phi, \theta, \psi)$ vector of rotation angles about x, y, z axes in $\{i\}$.
- $\mathscr{V}_r := \mathrm{col}(u_r, v_r, w_r)$: relative velocity
- $\mathscr{V}_c^i := \mathrm{col}(\mathscr{V}_{c,x}^i, \mathscr{V}_{c,y}^i, \mathscr{V}_{c,z}^i)$: wind velocity

Body kinematics:

$$\dot{\mathscr{X}}_{bi}^{i} = R_b^i(\Theta_{ib})\mathscr{V}_r + \mathscr{V}_c^i. \tag{1}$$

New reference frame $\{c\}$, obtained from the frame $\{b\}$ through

$$\begin{aligned} \alpha_c &= \arctan\left(\frac{w_r}{u_{rd}}\right) \\ \beta_c &= \arctan\left(\frac{v_r}{\sqrt{u_{rd}^2 + w_r^2}}\right), \end{aligned} \tag{2}$$

with u_{rd} the desired, constant, longitudinal relative speed

Kinematics

Objective

Control relative orientation of the $\{c\}$ frame with respect to the $\{f\}$ via $\Theta_{fc} := \mathrm{col}(\phi_c, \theta_c, \psi_c)$ (equiv. $R_f^c(\Theta_{bc})$) such that

$$\lim_{t\to\infty} \mathscr{X}_{bf}^{f}(t) := \mathrm{col}(x_{bf}, y_{bf}, z_{bf}) = 0.$$

Path: $\Theta_{if} = \mathrm{col}(\phi_f, \theta_f, \psi_f)$, curvature κ, torsion τ (known)
Serret-Frenet dynamics:

$$\begin{bmatrix} \dot{T} \\ \dot{N} \\ \dot{B} \end{bmatrix} = \dot{s} J \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{3}$$

with $\dot{s}$ instateneous speed of virtual UAV on the path and

$$J := \begin{bmatrix} 0 & -\kappa(s) & 0 \\ \kappa(s) & 0 & -\tau(s) \\ 0 & \tau(s) & 0 \end{bmatrix}. \tag{4}$$

$$
\begin{aligned}
\dot{\mathscr{X}}_{bf} &= R_b^f(\Theta_{bf})\mathscr{V}_{bi}^b - \dot{s}e_1 - \dot{s}J\mathscr{X}_{bf} \\
&= R_b^f(\Theta_{bf})(\mathscr{V}_r - R_i^b(\Theta_{ib})\mathscr{V}_c^i) - \dot{s}e_1 - \dot{s}J\mathscr{X}_{bf} \\
&= R_b^f(\Theta_{bf})\mathscr{V}_r - \dot{s}e_1 - \dot{s}J\mathscr{X}_{bf} + R_i^f(\Theta_{if})\mathscr{V}_c^i,
\end{aligned} \tag{5}
$$

Useful identities: $a\sin x + b\cos x = \sqrt{a^2+b^2}\sin(x + \arctan(\frac{b}{a})) = \sqrt{a^2+b^2}\cos(x - \arctan(\frac{b}{a}))$,

$$
\begin{aligned}
R_b^f(\Theta_{bf})\mathscr{V}_r &= R_c^f(\Theta_{fc})R_b^c(\Theta_{bc})\mathscr{V}_r \\
&= R_c^f(\Theta_{fc})\begin{bmatrix} V_2 c(\lambda) \\ V_2 s(\lambda) \\ V_1 s(\gamma) \end{bmatrix} = \begin{bmatrix} V_3 c(\mu) \\ V_4 s(\zeta) \\ V_4 s(\zeta) \end{bmatrix}
\end{aligned} \tag{6}
$$

with $c(\cdot) := \cos(\cdot)$, $s(\cdot) := \sin(\cdot)$

Kinematics

New variables:

$$
\begin{aligned}
V_1 &:= \sqrt{u_r^2 + w_r^2} && (7)\\
\gamma &:= \alpha_c - \arctan(\frac{w_r}{u_r}) && (8)\\
V_2 &:= \sqrt{(V_1 \cos\gamma)^2 + v_r^2} && (9)\\
\lambda &:= \beta_c - \arctan(\frac{v_r}{V_1 \cos\gamma}) && (10)\\
V_3 &:= \sqrt{(V_2 \cos(\lambda - \psi_c))^2 + (V_1 \sin\gamma)^2} && (11)\\
\mu &:= \theta_c + \arctan(\frac{V_1 \sin\gamma}{V_2 \cos(\lambda - \psi_c)}) && (12)\\
V_4 &:= \sqrt{(V_3 \sin\mu)^2 + (V_2 \sin(\lambda - \psi_c))^2} && (13)\\
\zeta &:= \phi_c - \arctan(\frac{V_3 \sin\mu}{V_2 \sin(\lambda - \psi_c)}). && (14)
\end{aligned}
$$

Kinematics

Assumption (A.1)

The relative longitudinal velocity is non-zero and positive, i.e. $u_r > 0$.

Assumption (A.2)

The roll is negligible, i.e. $\phi = 0$.

$$\begin{bmatrix} \dot{x}_{bf} \\ \dot{y}_{bf} \\ \dot{z}_{bf} \end{bmatrix} = \begin{bmatrix} V_3 \cos\mu \\ -V_2 \sin(\lambda - \psi_c) \\ V_3 \sin\mu \end{bmatrix} - \begin{bmatrix} \dot{s} \\ 0 \\ 0 \end{bmatrix} - \dot{s} J \begin{bmatrix} x_{bf} \\ y_{bf} \\ z_{bf} \end{bmatrix} + \begin{bmatrix} V_x^f \\ V_y^f \\ V_z^f \end{bmatrix}. \quad (15)$$

Assumption (A.3)

The wind is considered either constant or slowly time-varying, and irrotational, i.e., $\dot{\mathscr{V}}_c^i = 0$.

Outline

Wind estimator

Proposition

Consider the kinematic model (1) that is perturbed through the wind velocity $\mathscr{V}_c^i$ *that satisfies A.3. Then, the estimator*

$$\dot{\xi} = -\frac{\partial \beta}{\partial \mathscr{X}_{bi}^i}(R_b^i(\Theta_{ib})\mathscr{V}_r + \xi + \beta(\mathscr{X}_{bi}^i)), \tag{16}$$

with

$$\beta(\mathscr{X}_{bi}^i) := \Sigma \mathscr{X}_{bi}^i, \qquad \Sigma = diag(\sigma_i), \sigma_i > 0, i = \{1, 2, 3\}. \tag{17}$$

provides a globally exponentially convergent estimate of the wind velocity given by $\hat{\mathscr{V}}_c^i := \xi + \beta(\mathscr{X}_{bi}^i)$.

Proof.

$$\dot{\mathscr{Z}}_c^i = \dot{\xi} + \frac{\partial \beta}{\partial \mathscr{X}_{bi}^i}\dot{\mathscr{X}}_{bi}^i = -\frac{\partial \beta}{\partial \mathscr{X}_{bi}^i}\mathscr{Z}_c^i, \tag{18}$$

$V_z(\mathscr{Z}_c^i) = |\mathscr{Z}_c^i|$

$$\dot{V}_z \leq -\min(\sigma_i)|\mathscr{Z}_c^i|, \tag{19}$$

Outline

Integral input-to-state stability (iISS)

Theorem (Lyapunov characterization of iISS (Angeli,2000))

Consider the system

$$\dot{x} = f(x, u), \tag{20}$$

where $f : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ *is a locally Lipschitz function with the state* $x \in \mathbb{R}^{n_x}$*, and the input signals* $u : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ *that are considered to be locally essentially bounded. System (20) is said to be iISS if there exists a positive definite radially unbounded continuously differentiable function* V*, a* $\mathcal{K}$ *function* γ *and a* $\mathcal{PD}$ *function* α *satisfying for all* $x \in \mathbb{R}^{n_x}$ *and all* $u \in \mathbb{R}^m$

$$\frac{\partial V_x(x)}{\partial x} f(x, u) \leq -\alpha_x(|x|) + \gamma(|u|). \tag{21}$$

Theorem (iISS of cascaded systems (Chaillet,2008))

Consider the cascaded system

$$\dot{x} = f(x,z) \quad (22a)$$
$$\dot{z} = g(z), \quad (22b)$$

with $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_x}$, $g : \mathbb{R}^{n_z} \to \mathbb{R}^{n_z}$ *locally Lipschitz with the property that* $f(0) = 0$, $g(0) = 0$. *Assume that the origin of (22b) is globally attractive and that there exist* $\varepsilon > 0$, *two continuous, positive definite, radially unbounded functions* V_x, V_z, *differentiable on* $\mathbb{R}^{n_x}$ *and* $\mathcal{B}_\varepsilon \setminus 0$ *respectively, and satisfying*

$$\frac{\partial V_x(x)}{\partial x} f(x,z) \leq -\alpha_x(|x|) + \gamma(|z|), \quad \forall \in \mathbb{R}^{n_x} \quad (23)$$

$$\frac{\partial V_z(z)}{\partial z} g(z) \leq -\alpha_z(|z|), \quad \forall z \in \mathcal{B}_\varepsilon \setminus 0, \quad (24)$$

where $\alpha_x, \alpha_z \in \mathcal{PD}$ *and* $\gamma \in \mathcal{K}$. *Then under the condition that* $\gamma(s) = \mathcal{O}(\alpha_z(s))$ *in a neighbourhood of zero, the cascade (22) is GAS.*

Proposition

Consider the differential kinematic model (15) along with the observer in (16)-(17). Then, the guidance laws given by

$$\mu = -\arctan(\frac{k_3 z_{bf} + \rho_1}{\rho_2}) \tag{25}$$

$$\rho_1 = \frac{(\hat{\mathscr{V}}^f_{c,z})^2 k_3 z_{bf} - |\hat{\mathscr{V}}^f_{c,z}|_1 \sqrt{V_3^2 k_3^2 z_{bf}^2 + (V_3^2 - (\hat{\mathscr{V}}^f_{c,z})^2)\rho_2^2}}{V_3^2 - (\hat{\mathscr{V}}^f_{c,z})^2} \tag{26}$$

$$\psi_c = \lambda - \arctan(\frac{k_2 y_{bf} + \rho_3}{\rho_4}) \tag{27}$$

$$\rho_3 = \frac{(\hat{\mathscr{V}}^f_{c,y})^2 k_2 y_{bf} - |\hat{\mathscr{V}}^f_{c,y}|_1 \sqrt{V_2^2 k_2^2 y_{bf}^2 + (V_2^2 - (\hat{\mathscr{V}}^f_{c,y})^2)\rho_4^2}}{V_2^2 - (\hat{\mathscr{V}}^f_{c,y})^2} \tag{28}$$

$$\dot{s} = \frac{k_1 x_{bf}}{\sqrt{x_{bf}^2 + \rho_0^2}} + V_3 \cos\mu + \hat{\mathscr{V}}^f_{c,x} \tag{29}$$

with $k_1, k_2, k_3, \rho_0, \rho_2, \rho_3 > 0$*, ensure that* $\mathscr{X}_{bf} = 0$ *is GAS.*

Proof (Sketch)

Lyapunov fct: $V_x(\mathscr{X}_{bf}) := \ln(1 + |\mathscr{X}_{bf}|^2)$

$$\begin{aligned}
\dot{V}_x \quad &\le \quad \frac{1}{1+|\mathscr{X}_{bf}|^2}\Big(-\frac{k_1 x_{bf}^2}{\sqrt{x_{bf}^2+\rho_0^2}} - \frac{V_2 k_2 y_{bf}^2}{\sqrt{(k_2 y_{bf}+\rho_3)^2+\rho_4^2}} - \frac{V_3 k_2 z_{bf}^2}{\sqrt{(k_2 y_{bf}+\rho_1)^2+\rho_2^2}} \\
&- \quad y_{bf}\Big(\frac{V_2\rho_3}{\sqrt{(k_2 y_{bf}+\rho_3)^2+\rho_4^2}} - V_{c,y}^f\Big) - z_{bf}\Big(\frac{V_3\rho_1}{\sqrt{(k_3 z_{bf}+\rho_1)^2+\rho_2^2}} - V_{c,z}^f\Big) \\
&- \quad x_{bf}\mathscr{Z}_{c,x}^i\Big) \\
&\le \quad -\frac{1}{1+|\mathscr{X}_{bf}|^2}\Big(\frac{k_1 x_{bf}^2}{\sqrt{x_{bf}^2+\rho_0^2}} + \frac{V_2 k_2 y_{bf}^2}{\sqrt{k_2^2 y_{bf}^2+\rho_3^2+\rho_4^2}} + \frac{V_3 k_3 z_{bf}^2}{\sqrt{k_3^2 z_{bf}^2+\rho_1^2+\rho_2^2}}\Big) \\
&+ \quad |\mathscr{Z}_c^i|.
\end{aligned} \tag{30}$$

$$\frac{V_2\rho_3}{\sqrt{(k_2 y_{bf}+\rho_3)^2+\rho_4^2}} \equiv \hat{V}_{c,y}^f := R_i^f \hat{V}_{c,y}^i \tag{31}$$

$$\frac{V_3\rho_1}{\sqrt{(k_1 z_{bf}+\rho_1)^2+\rho_2^2}} \equiv \hat{V}_{c,z}^f := R_i^f \hat{V}_{c,z}^i. \tag{32}$$

Outline

Objective: Follow a clockwise helix moving with radius $R = 100m$ and a vertical separation of $h = 50m$ between helix loops

Helix parameterization: $x_f(s) = Rc(\frac{s}{\sqrt{R^2+(\frac{h}{2\pi})^2}})$, $y_f(s) = Rs(\frac{s}{\sqrt{R^2+(\frac{h}{2\pi})^2}})$,

$z_f(s) = \frac{h}{2\pi}\frac{s}{\sqrt{R^2+(\frac{h}{2\pi})^2}}$, $\phi_f(s) = 0$, $\theta_f(s) = \arctan(\frac{\frac{h}{2\pi}}{R})$,

$\psi_f(s) = \frac{sc(\theta_f(s))}{R} + \frac{\pi}{2}$, $\kappa(s) = \frac{R}{\sqrt{R^2+(\frac{h}{2\pi})^2}}$, $\tau(s) = \frac{\frac{h}{2\pi}}{\sqrt{R^2+(\frac{h}{2\pi})^2}}$.

- $\mathscr{X}_{bf}(0) = (-10, 100, 10)(\mathrm{m})$
- $u_r = v_r = w_r = 10\frac{\mathrm{m}}{\mathrm{sec}}$
- $V_x^i = -5, V_y^i = -5, V_z^i = 1(\frac{\mathrm{m}}{\mathrm{sec}})$
- $k_i = 1$
- $\xi(0) = \Sigma\mathscr{X}_{bi}^i(0), \sigma_i = 1.$

No disturbances

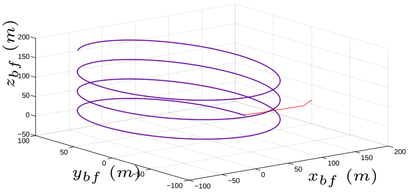

Figure: Desired (blue) and actual (red) paths

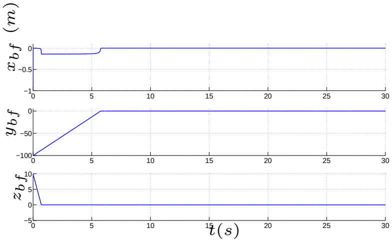

Figure: Body position with respect to the Serret-Frenet frame

With disturbances

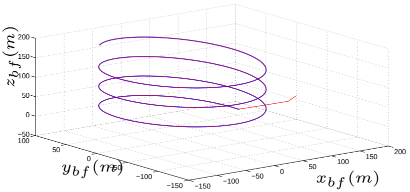

Figure: Desired (blue) and actual (red) paths

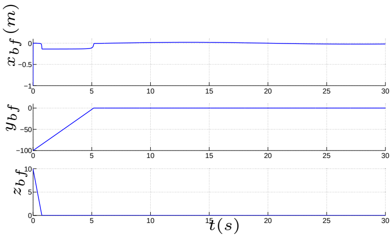

Figure: Body position with respect to the Serret-Frenet frame

Wind velocity estimation

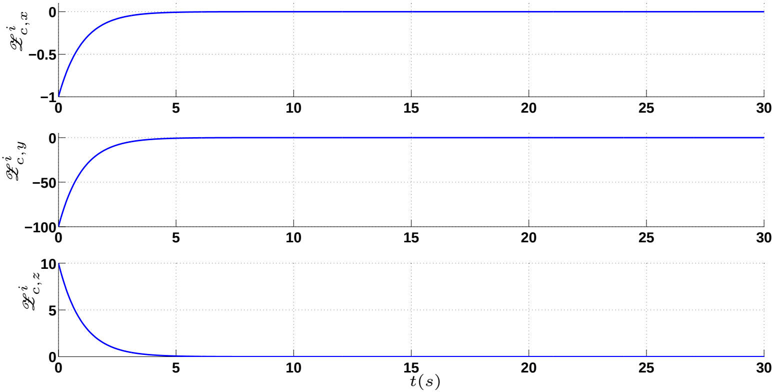

Figure: Wind velocity estimation errors (ms^{-1})