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Control of thrust-propelled underactuated vehicles. Application to VTOL vehicles.

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Projet ANR ”SCUAV”

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Thrust-propelled underactuated vehicles

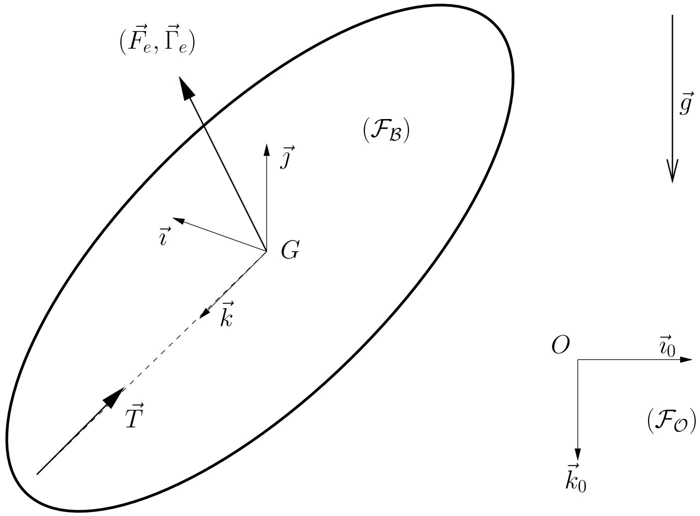
- Thrust force in a direction linked to the vehicle's main body
- Actuators producing force/torques to rotate the body (complete orientation actuation)

Examples :

1. In the plane (SE(2)): boats, hovercrafts, (blimps, PVTOL)
2. In 3D space (SE(3)): submarines, aeroplanes, blimps, rockets, VTOL vehicles (helicopters, Xflyers, Bertin's HoverEye,...)

Objective : develop the basics of a unifying approach to the control of these systems

Modeling equations for control design



$e_3 = (0,0,1)^T$, $u = \frac{T}{m}$, $\gamma_e = \frac{F_e}{m}$

R : rotation matrix between $\mathcal{F}_{\mathcal{O}}$ and $\mathcal{F}_{\mathcal{B}}$

$\vec{OG} = (\vec{\imath}_0\ \vec{\jmath}_0\ \vec{k}_0)x$

$\frac{d}{dt}\vec{OG} = (\vec{\imath}\,\vec{\jmath}\,\vec{k})v$

$\vec{\omega} = (\vec{\imath}\,\vec{\jmath}\,\vec{k})\omega$

$$\begin{pmatrix} \dot{x} \\ \dot{v} \\ \dot{R} \end{pmatrix} = \begin{pmatrix} Rv \\ -\omega^\wedge v - ue_3 + R^T\gamma_e(\dot{x}, \ddot{x}, R, \omega, \dot{\omega}, t) \\ R\omega^\wedge \end{pmatrix}$$

$$J\dot{\omega} = -\omega^\wedge J\omega + \Gamma + \Gamma_e(\dot{x}, \ddot{x}, R, \omega, \dot{\omega}, t)$$

Control problems from joystick augmentation to automatic flight

1. Thrust direction control (joystick augmentation mode, level 1)

2. Velocity control (joystick augmentation mode, level 2)

3. Position control, trajectory tracking (automatic flight)

For control purposes, minimal parametrizations of orientations (like Euler parameters) should be banished. Use rotation matrices or quaternions.

Simplifying assumptions

1. Γ_e can always be “dominated “ by the control torque Γ so that ω can be used as an intermediary control variable (backstepping technique)

(for problems 2 and 3 :)

2. F_e depends on $\dot{x}$ and t only
   - example: the body is a dense sphere
   - counter-example: planes with lift forces depending on attack angles
3. $\|F_e(\dot{x},t)\| \leq c_1 + c_2\|\dot{x}\|^2$
4. $\dot{x}^T F_e(\dot{x},t) \leq c_3\|\dot{x}\| - c_4\|\dot{x}\|^3$ (passivity property)
5. Complete measurement of the state (x, R, v, ω)
6. F_e is either measured or estimated

Thrust direction control

- $\gamma(t) \in \mathbb{R}^3$: unit vector giving the desired thrust direction at time t
- $\bar{\gamma} := R^T \gamma$
- Objective : exponential stabilization of $\bar{\gamma} - e_3 = 0$ ($\Leftrightarrow$ exp. stab. of $\theta = 0$ with $\cos(\theta) = \bar{\gamma}_3$)

System :

$$\dot{R} = R\omega^\wedge$$

Control :

$$\begin{cases} \omega_1 = -\frac{k\bar{\gamma}_2}{(1+\bar{\gamma}_3)^2} - \gamma^T e_1^\wedge \dot{\gamma} \\ \omega_2 = -\frac{k\bar{\gamma}_1}{(1+\bar{\gamma}_3)^2} - \gamma^T e_2^\wedge \dot{\gamma} \end{cases} \qquad k > 0,\ e_1 = (1,0,0)^T,\ e_2 = (0,1,0)^T$$

$\omega_3(t)$ is free and the domain of attraction is $(-\pi, \pi)$.

Velocity control (1/2)

- $\dot{x}_r$: desired velocity of G
- $\tilde{v} := R^T(\dot{x} - \dot{x}_r)$
- $\gamma(\dot{x}, t) = \gamma_e(\dot{x}, t) - \ddot{x}_r(t) \qquad (\gamma_e = \frac{F_e}{m})$

Error system

$$\begin{cases} \dot{\tilde{v}} = -\omega^\wedge \tilde{v} - u e_3 + R^T \gamma \\ \dot{R} = R\omega^\wedge \end{cases}$$

The equilibrium $\tilde{v} = 0$ ($\Rightarrow u e_3 = R^T \gamma$) defines a (locally) unique thrust direction only if $\gamma \neq 0$

$\longrightarrow$ Assumption : $\gamma(\dot{x}_r(t), t) \neq 0 \ , \ \forall t$

- Necessary for the existence of "classical" control solutions
- Ex: hovering VTOL vehicle ($\dot{x}_r = 0$) submitted to gravity $\Rightarrow \gamma(\dot{x}_r(t), t) = g e_3$ ($\neq 0$)
- Counter-ex : Boat at rest ($\dot{x}_r = 0$), no current ($\gamma_e = 0$) $\Rightarrow \gamma(\dot{x}_r(t), t) = 0$

Velocity Control (2/2)

Control 1 :

$$\left\{\begin{array}{lcl} u & = & \bar{\gamma}_3 + \|\gamma\| k_1 \tilde{v}_3 \\ \omega_1 & = & -\|\gamma\| k_2 \tilde{v}_2 - \frac{k_3 \|\gamma\| \bar{\gamma}_2}{(\|\gamma\| + \bar{\gamma}_3)^2} - \frac{1}{\|\gamma\|^2} \gamma^T (Re_1)^\wedge \dot{\gamma} \\ \omega_2 & = & \|\gamma\| k_2 \tilde{v}_1 - \frac{k_3 \|\gamma\| \bar{\gamma}_1}{(\|\gamma\| + \bar{\gamma}_3)^2} - \frac{1}{\|\gamma\|^2} \gamma^T (Re_2)^\wedge \dot{\gamma} \end{array}\right. \qquad (\ k_{1,2,3} > 0)$$

Control 2 : includes a complementary integral action

Same control expression with $\gamma := \gamma_e - \ddot{x}_r + h(\|I_v\|^2) I_v$

$I_v = \int_0^t (\dot{x}(s) - \dot{x}_r(s)) ds + I_0$

$h\ (\mathbb{R}^+ \to \mathbb{R}^+)$: bounded fct. such that

- $|h(s^2)s| < \eta\ ,\ \forall s$
- $0 < \frac{d}{ds}(h(s^2)s) < \beta\ ,\ \forall s$

Ex: $h(s) = \frac{\eta}{\sqrt{(1+s)}}$

Position control - Trajectory tracking

- $x_r(t)$: desired (or reference) position at time t
- $\tilde{x} := x - x_r$

Control 1 : the previous velocity control 2, since $\tilde{x}(t) = I_v(t)$ with $I_0 = 0$

Control 2 : includes a position integral term z calculated as the solution to

$$\ddot{z} = -2k_z\dot{z} - k_z^2(z - sat_\Delta(z)) + k_z h_z(\|\tilde{x}\|^2)\tilde{x} \qquad (k_z > 0,\ z(0) = \dot{z}(0) = 0)$$

Same control expression with $\gamma := \gamma_e - \ddot{x}_r + h(\|\tilde{x} + z\|^2)(\tilde{x} + z) + \ddot{z}$ and $\tilde{v}$ replaced by $\bar{v} := \tilde{v} + R^T\dot{z}$

Control 3 : control 2 modified to ensure a constant sign thrust control u (as required for some systems)

+ robustification adjustments w.r.t. situations when $\|\gamma\|$ crosses zero, or becomes small

Trajectory tracking simulations

- No aerodynamical forces, gravity only
  (robustness w.r.t. system modeling errors)
- Aerodynamical forces, no wind
  (robustness w.r.t. system + environment modeling errors)
- Aerodynamical forces, strong wind gusts
  (robustness w.r.t. environmental perturbations)
- Important initial errors (size of the operating domain)
- Even larger initial errors (interception capabilities)

Possible extensions

- Case where F_e depends also on the vehicle's attitude (aeroplanes)
- Get rid of the assumption $\gamma \neq 0 \rightarrow$ application of non-classical control techniques (transverse function approach,...)
- Complement works on state estimation (multisensory fusion)
- Pursue works on the on-line estimation of F_e (in relation to the previous issue)