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Robust 3D path planning based on fractional attractive force for mobile robot and UAV

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IMS
Automatic control Group/Crone team
SYMM Project
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1 - Introduction

- IPB/ENSEIRB-MATMECA
- IMS
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1 - Introduction

Context *The path planning design consists in the elaboration of a strategy to reach a target.*

Potential *fields introduce force constraints* ***to ensure curvature continuity*** *of trajectories and thus to facilitate path-tracking design.*

Problem

Dynamic environment

Previous works

- Fractional repulsive potential: to avoid fixed/mobile obstacles
- Danger level of each obstacle was characterized by the fractional differentiation order
- Road was determined by ***taking into account danger of each obstacle***.
- ***Dynamic obstacles***: the method was extended to obtain trajectories by considering repulsive and attractive (Ge and Cui method) potentials taking into account position and velocity of the robot, target and obstacles.

Objective

But, in case of ***robot or UAV parameter variations***, these two previous attractive forces do not allow to obtain ***robust path planning***.

1 - Introduction

Methodology

A new fractional based attractive force

Expected result

Robust path planning of mobile robot or UAV in dynamic environment

The objective of our work in this paper:

To define an attractive force
To study the robustness of the approach
To apply the approach in 2D dynamic environment
To apply the approach in 3D dynamic environment

2 - Fractional mathematical background

2 - Fractional mathematical background

2.1 - Fractional integration

The fractional integral of a function *f(t)* is defined by:

$$(I_{a^+}^n f)(t) \stackrel{\Delta}{=} \frac{1}{\Gamma(n)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-n}} d\tau$$

where $t > a$ and n is the real positive integration order, $\Gamma(n)$ is the Euler Gamma function:

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx.$$

The Laplace transform of the integral of a function *f(t)* is:

$$\mathrm{L}\left\{I_0^n f(t)\right\} = \int_0^\infty \mathrm{e}^{-pt} \left(\frac{1}{\Gamma(n)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-n}} d\tau \right) dt = \frac{1}{s^n} F(s),$$

where F(s) is the Laplace transform of *f(t)*.

2 - Fractional mathematical background

2.2 - Fractional differentiation

The Riemann-Liouville fractional derivative of order *n* of *f(t)* is defined as:

$$D_{t_0}^{n} f(t) \triangleq \left(\frac{d}{dt}\right)^{n+1} \left(I_{t_0}^{1-n} f(t)\right).$$

Second definition (Grünwald's definition) is:

$$D_{t_0}^{n} f(t) = \lim_{h \to 0} \frac{1}{h^n} \sum_{j}^{(t-t0)/h} (-1)^j \binom{n}{j} f(t - jh)$$

where

$$\binom{n}{j} = \frac{\Gamma(n+1)}{\Gamma(j+1)\Gamma(n-j+1)}.$$

Global operator: ***the value of the fractional derivative function at t depends on the whole past of the function.***

The Laplace transform is: $L\left\{D_0^n f(t)\right\} = s^n F(s).$

3 - Fractional attractive force definition

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3.1 - Attractive force definition

Conventionally, the attractive potential is defined as a function of the relative distance between the robot and the target, only when the target is a fixed point in space.

The force applied on the robot is given by:

$$F_{rob} = m_{rob}.a_{rob}$$

with m_{rob} and a_{rob}, the robot mass and accelaration.

On another way, this force is given by:

$$F_{rob} = F_{tar} + F_{att}$$

with

$$F_{tar} = m_{tar}.a_{tar}$$

with F_{tar} the target attractive force,
m_{tar} and a_{tar}, the target mass and accelaration.

3 - Fractional attractive force definition

3.2 - Ge and Cui attractive force

The Ge and Cui method allows *to obtain trajectories in real time by considering repulsive and attractive potentials* taking into account position and velocity of the robot with respect to obstacles.

The Ge and Cui virtual attractive force is defined by:

$$F_{att} = \alpha_p.(X_{tar} - X_{rob}) + \alpha_v.(V_{tar} - V_{rob})$$

➡ How to determine these parameters, constraints / mass variations?

➡ Dynamic analysis allows to interpret the influence of the parameters

So by taking m_{tar} equal to m_{rob}:

$$m_{rob}.(a_{tar} - a_{rob}) + \alpha_v.(V_{tar} - V_{rob}) + \alpha_p.(X_{tar} - X_{rob}) = 0.$$

with

$$\begin{cases} e(t) = X_{tar} - X_{rob} \\ \dfrac{de(t)}{dt} = V_{tar} - V_{rob} \\ \dfrac{d^2e(t)}{dt^2} = a_{tar} - a_{rob} \end{cases}$$

3 - Fractional attractive force definition

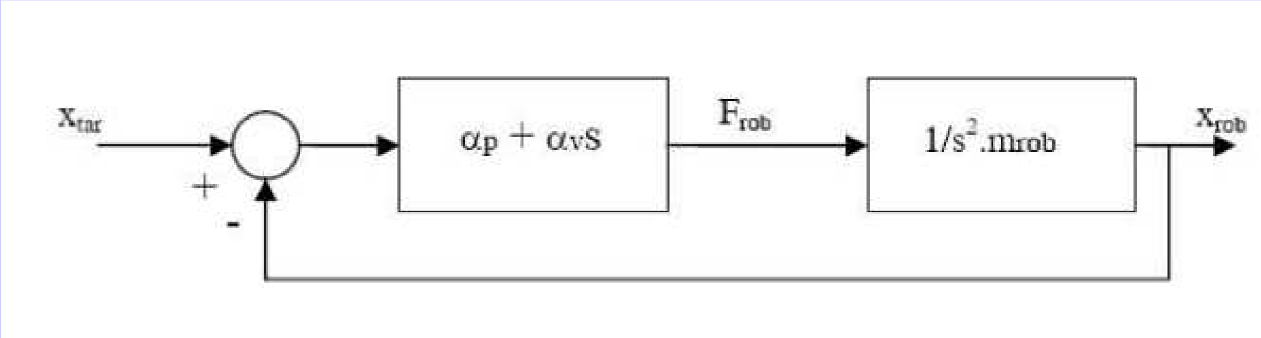
Differential equation

$$\frac{d^2e(t)}{dt^2} + \frac{\alpha_v}{m_{rob}} \cdot \frac{de(t)}{dt} + \frac{\alpha_p}{m_{rob}} e(t) = 0.$$

In Laplace domain

$$s^2E(s) + \frac{\alpha_v}{m_{rob}} sE(s) + \frac{\alpha_p}{m_{rob}} E(s) = 0.$$

This can be interpreted by a classical control scheme where ***α_p and α_v are the parameters of a PD controller***:



where α_p and α_v are the parameters of a ***clasical PD controller***.

3 - Fractional attractive force definition

The corresponding open loop transfer function $\beta(s)$ is given by:

$$\beta(s) = \frac{\alpha_v s + \alpha_p}{m_{rob} s^2}.$$

The closed loop transfer function $H(s)$ is deduced:

$$H(s) = \frac{\beta(s)}{1 + \beta(s)},$$

lading to:

$$H(s) = \frac{(\frac{\alpha_v}{\alpha_p})s + 1}{(\frac{m_{rob}}{\alpha_p})s^2 + (\frac{\alpha_v}{\alpha_p})s + 1}.$$

The characteristic equation is deduced:

$$E_c(s) = \frac{s^2}{w_n^2} + \frac{2\xi}{w_n}s + 1 = 0,$$

with:

$$\begin{cases} w_n = \sqrt{\frac{\alpha_p}{m_{rob}}} \\ \xi = \frac{\alpha_v}{2\sqrt{\alpha_p m_{rob}}}. \end{cases}$$

The damping factor is dependent of the mass robot m_{rob}.
So the obtained ***trajectory is not robust*** in front of the mass robot variations.

3 - Fractional attractive force definition

So, the dynamic system behavior depends of the choice of the parameters α_p, α_v and m_{rob}

For a damping factor $\xi = \frac{\sqrt{2}}{2} = 0.707$, the robot mass is given by:

$$m_{rob} = \left(\frac{\alpha_v}{2.\xi}\right)^2 . \frac{1}{\alpha_p} = \frac{\alpha_v^2}{4\alpha_p.\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\alpha_v^2}{2\alpha_p}.$$

So, the condition to avoid oscillation $\xi > \frac{\sqrt{2}}{2}$, leads to the maximal mass defined by $m_{rob} \leq 0.5\frac{\alpha_v^2}{\alpha_p}$.

For example, for $m_{rob} = 1$, $\alpha_p = 0.005$, $\alpha_v = 0.1$ *(parameters values chosen in previous example by Ge and Cui to satisfied this relation)*

- So, this dynamic analysis allows to interpret the influence of the Ge and Cui parameters
- It also introduces methodology to determine these parameters

3 - Fractional attractive force definition

3.3 - Fractional attractive force

The proposed attractive force is based on velocity fractional derivative.

$$F_{att} = \alpha_p.(X_{tar} - X_{rob}) + \alpha_v.\frac{d^n(X_{tar} - X_{rob})}{dt^n}$$

where α_p and α_v are scalar positive parameters and ***n*** the fractional differentiation order.

So by taking m_{tar} equal to m_{rob}:

$$m_{rob}.(a_{tar} - a_{rob}) + \alpha_v.\frac{d^n(X_{tar} - X_{rob})}{dt^n} + \alpha_p.(X_{tar} - X_{rob}) = 0.$$

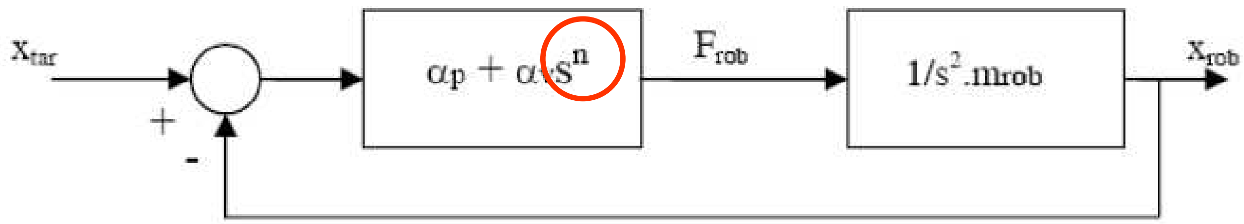
Differential equation

$$\frac{d^2 e(t)}{dt^2} + \frac{\alpha_v}{m_{rob}}.\frac{d^n(e(t))}{dt^n} + \frac{\alpha_p}{m_{rob}} e(t) = 0.$$

In Laplace domain, the relation becomes:

$$s^2 E(s) + \frac{\alpha_v}{m_{rob}} s^n E(s) + \frac{\alpha_p}{m_{rob}} E(s) = 0.$$

3 - Fractional attractive force definition



$$\beta(s) = \frac{\alpha_v s^n + \alpha_p}{m_{rob} s^2}.$$

where α_p and α_v are the parameters of a ***fractional PD controller***.

For $\omega \gg \omega_c = (\frac{\alpha_p}{\alpha_v})^{1/n}$, $\beta(s)$ can be approximated by:

$$\beta(s) \approx \frac{\alpha_v s^n}{m_{rob} s^2}$$

lading to:

$$\beta(s) = (\frac{\omega_{cg}}{s})^{n'}$$

with:

$$\begin{cases} n' = 2 - n \\ \omega_{cg} = (\frac{\alpha_v}{m_{rob}})^{\frac{1}{(2-n)}}. \end{cases}$$

3 - Fractional attractive force definition

The resonant factor and the damping factor can be deduced:

$$Q = \frac{1}{sin(2-n')\frac{\pi}{2}}$$

and

$$\xi(n') = -cos(\frac{\pi}{n'}).$$

- The damping factor is ***independent of the mass robot***
- This illustrates the ***robustness of the obtained trajectory.***

4 - Robustness Analysis

4 - Robustness analysis

Comparison of the open loop Nichols diagrams obtain with Ge and Cui and fractional methods

Parameters

m_{rob} is equal to [110, 150, 190, 250, 400]

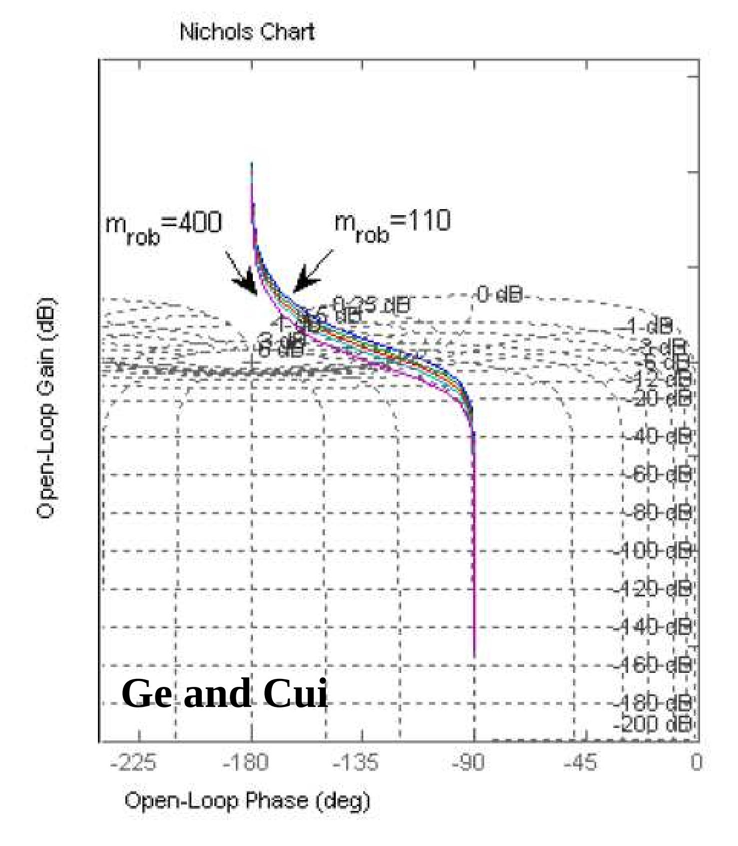
the nominal mass is 150 kg

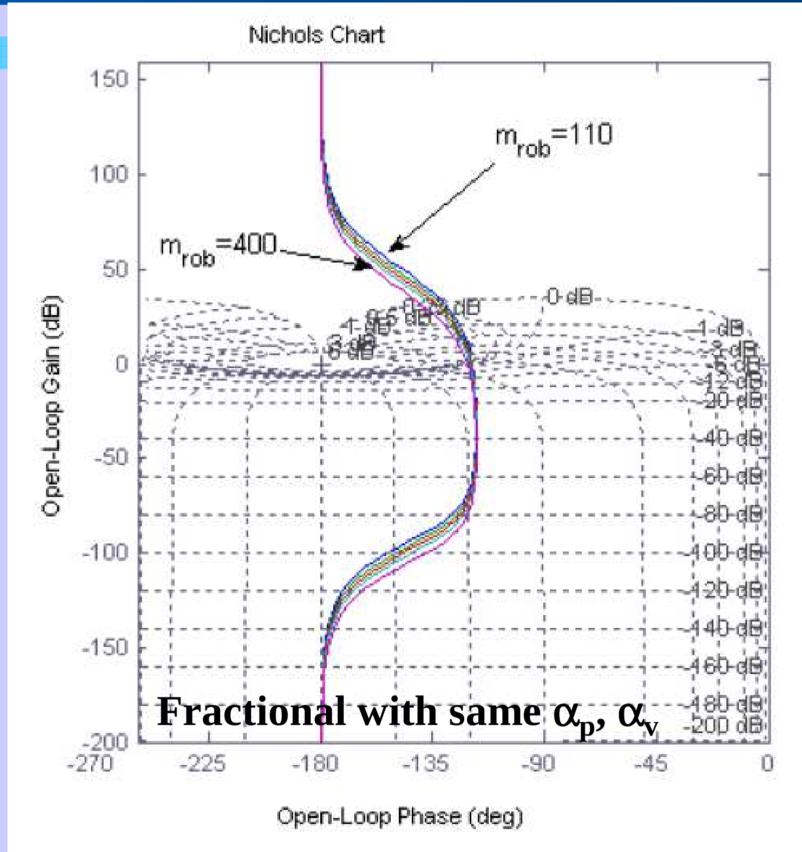
Ge and Cui parameters: α_p=0.002 and α_v= 0.8

fractional order n = 0.7

4 - Robustness analysis

Comparison with same α_p, α_v parameters





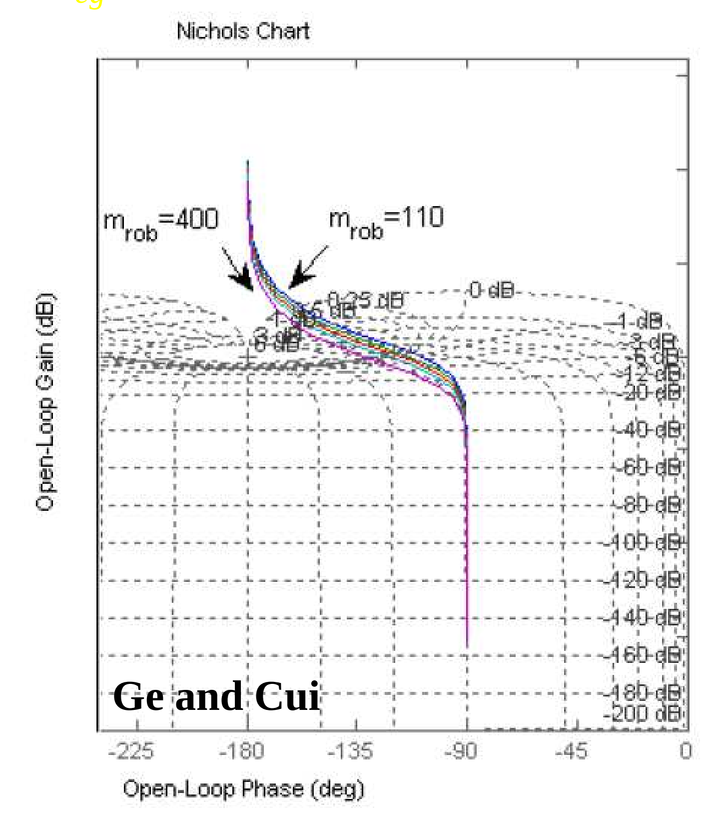
m_{rob} is varying from 110 to 400 kg
ω_{cg} from 0.0077 to 0.0027 rad/s
phase margin from 72◦ to 48◦
ξ from 0.85 to 0.45.

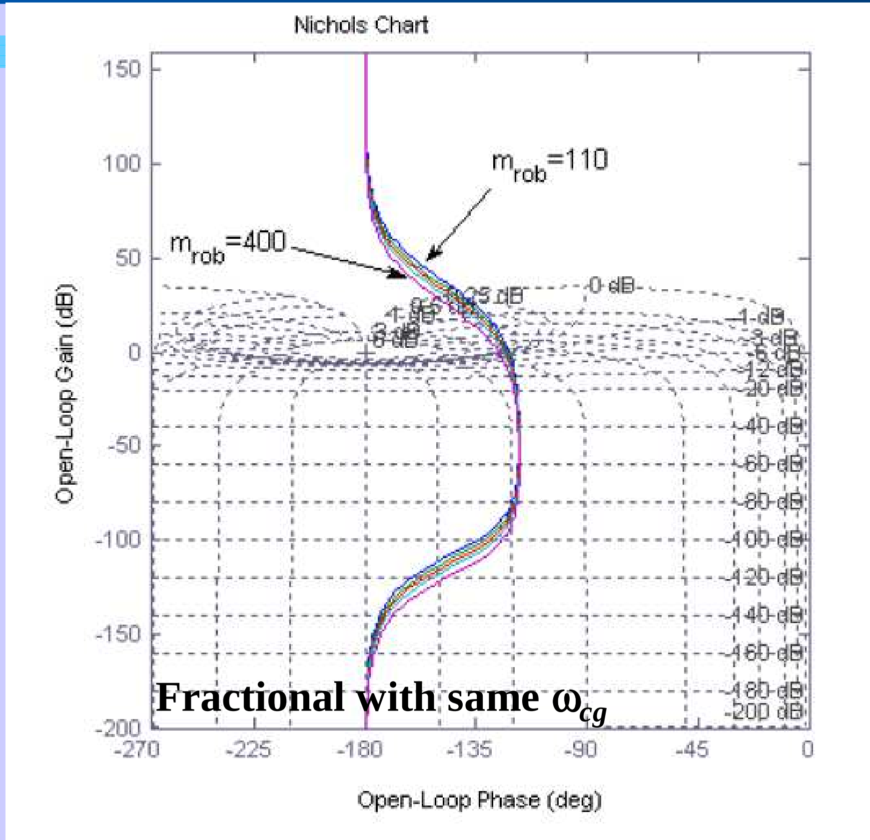
m_{rob} is varying from 110 to 400 kg
ω_{cg} from 0.023 to 0.0086 rad/s
phase margin from 61◦ to 59◦
$\xi = 0.7$.

The open loop is characterized by a vertical frequency template in the Nichols diagram, the phase margin is constant, and the stability degree and the damping factor are also constant.

4 - Robustness analysis

Comparison with same ω_{cg} than Ge and Cui for the nominal robot mass m_{rob}=150 kg, ω_{cg} =0.0058 rad/s





m_{rob} is varying from 110 to 400 kg,
ω_{cg} from 0.0077 to 0.0027 rad/s
phase margin from 72° to 48°
ξ from 0.85 to 0.45.

αp=0.002x0.22
αv= 0.8x0.22
m_{rob} is varying from 110 to 400 kg
ω_{cg} from 0.0073 to 0.0028 rad/s
phase margin from 59° to 54°
ξ = 0.7. → ***robust stability degree***

5 - Simulation results

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In this simulation, there is *not obstacle* in order to estimate the performances of the attractive forces.

Parameters

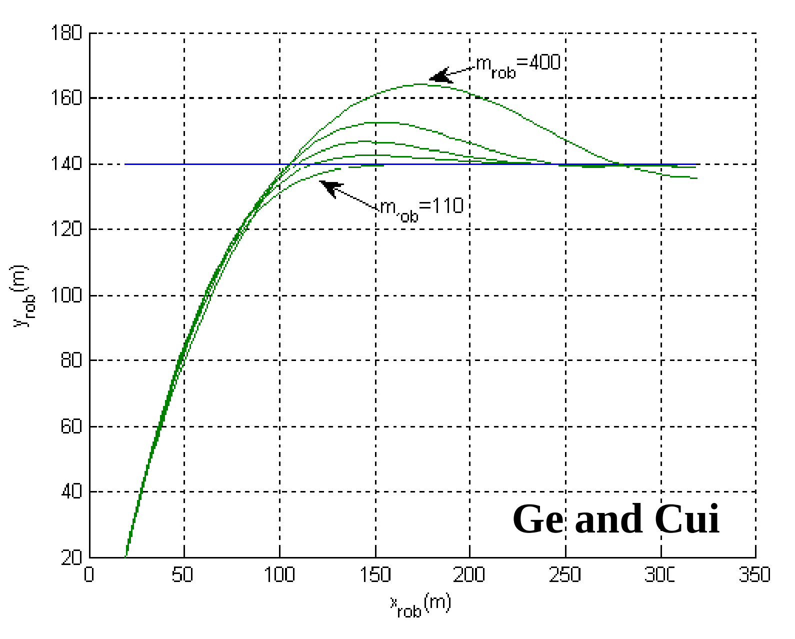
m_{rob} is equal to [110, 150, 190, 250, 400]

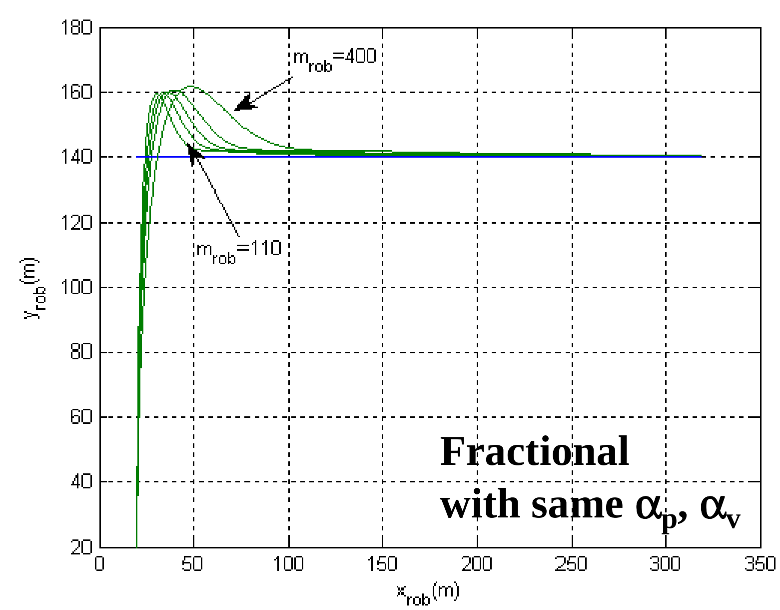
the nominal mass is 150 kg

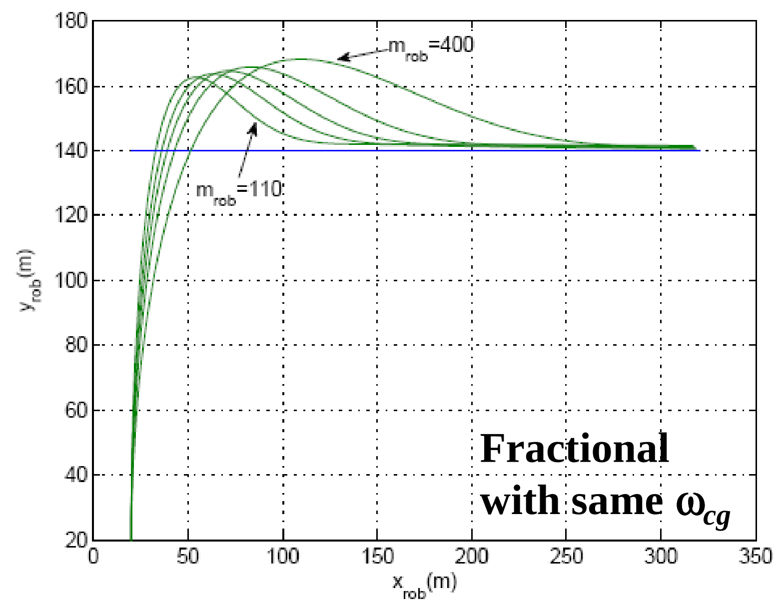
Ge and Cui parameters α_p=0.002 and α_v= 0.8

fractional order n = 0.7

5 - Simulation results

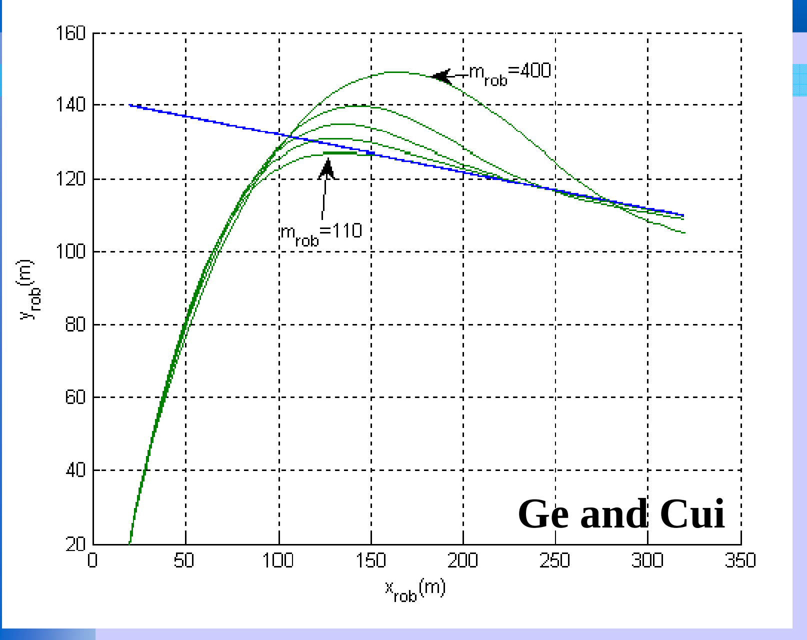


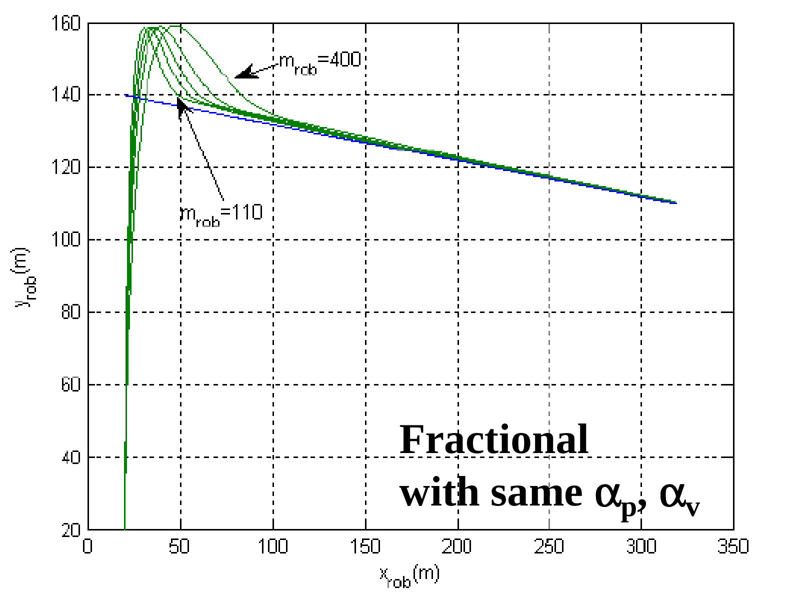


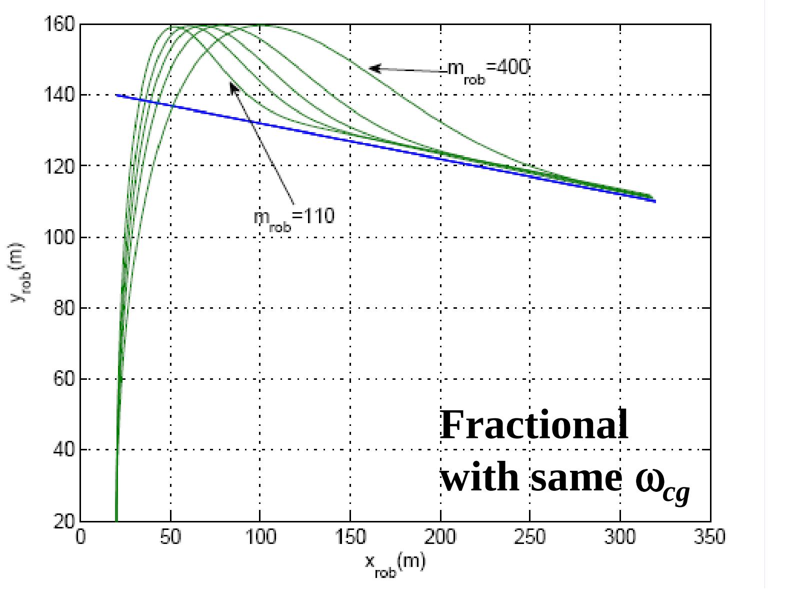


Robust and faster path planning despite robot mass variation

5 - Simulation results

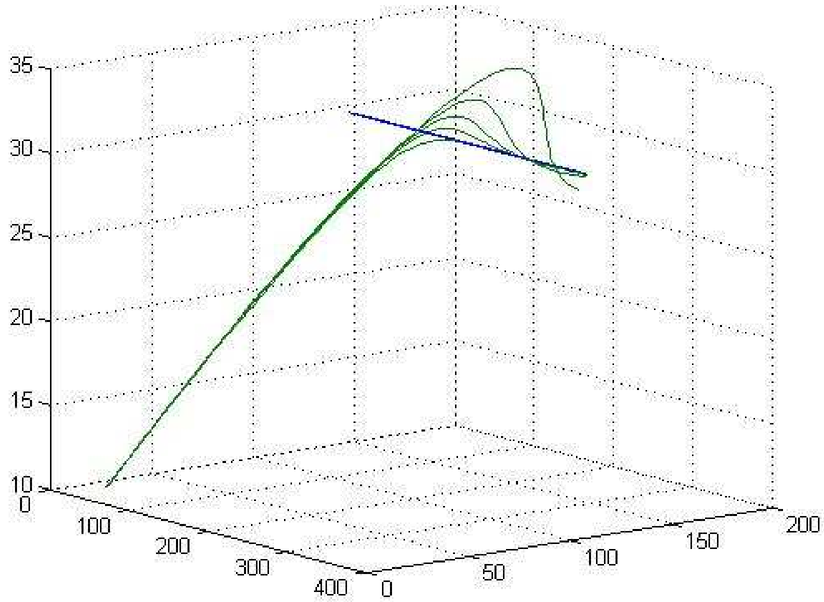




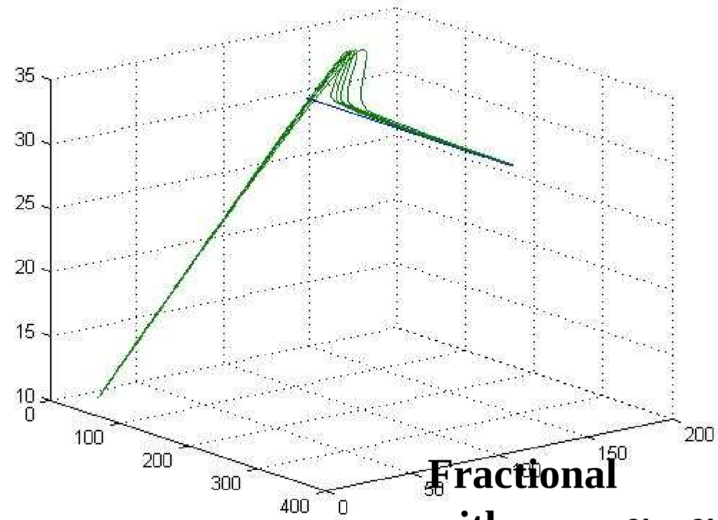


Robust and faster path planning despite robot mass variation

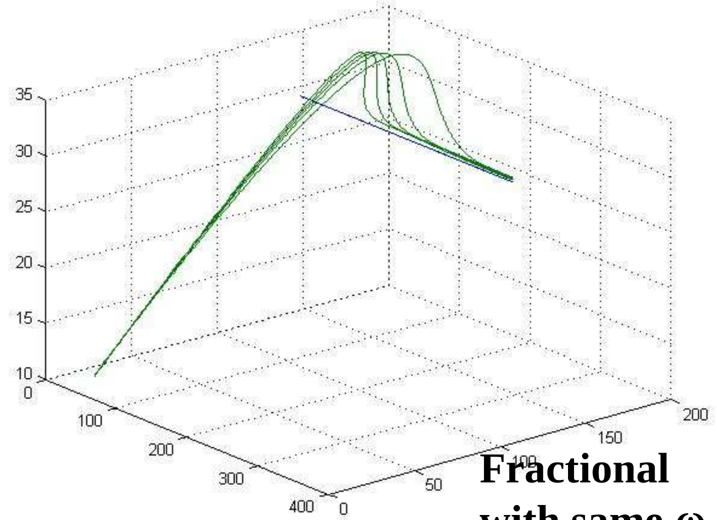
5 - 3D Simulation results



Ge and Cui



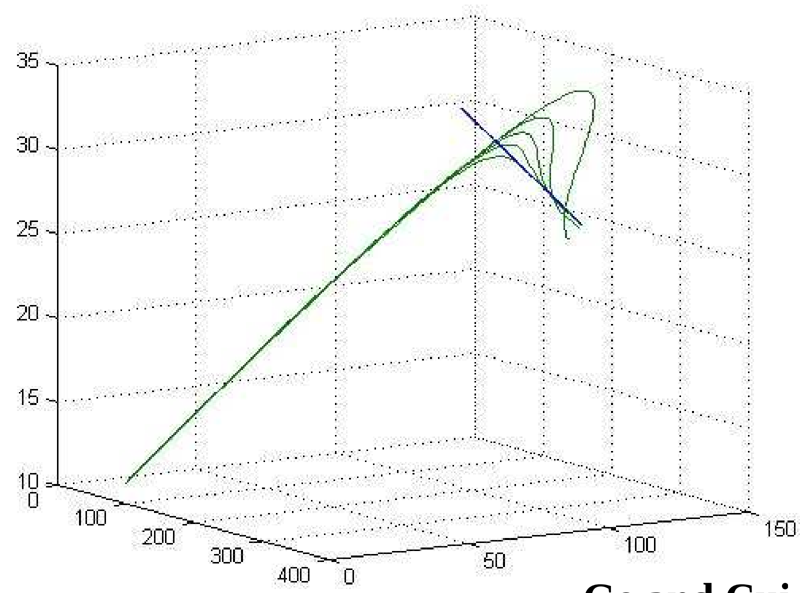
Fractional with same α_p, α_v



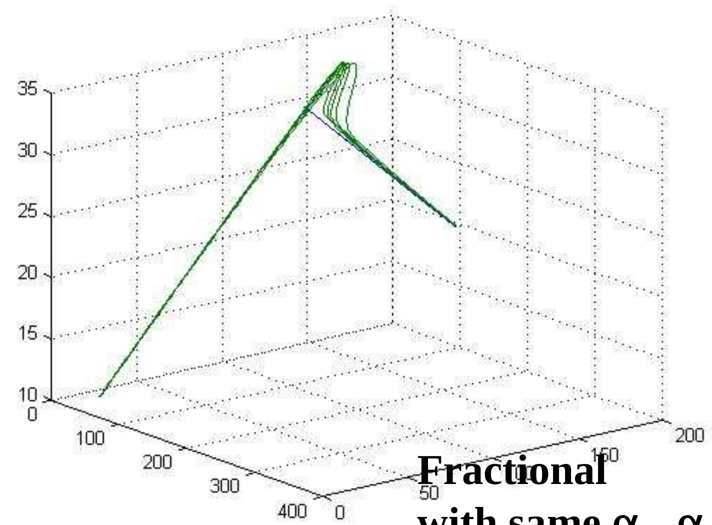
Fractional with same ω_{cg}

➡ ***Robust and faster path planning despite robot mass variation***

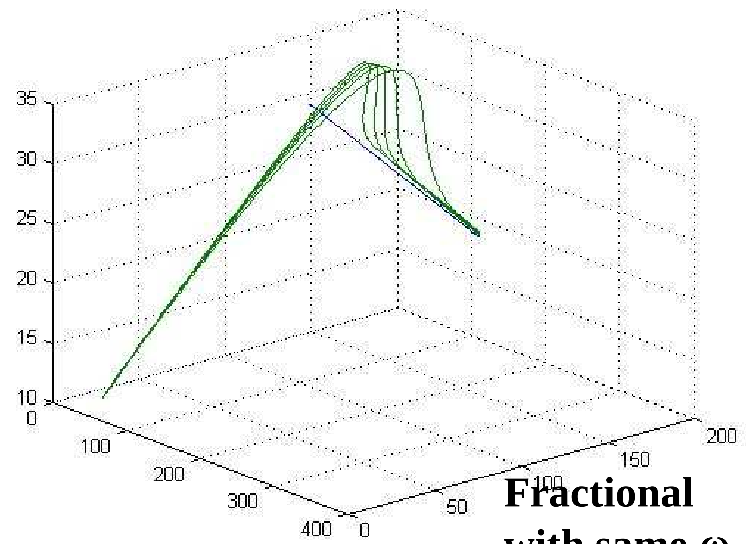
5 - 3D Simulation results



Ge and Cui



Fractional with same α_p, α_v



Fractional with same ω_{cg}

Robust and faster path planning despite robot mass variation

6 - Simulation results with obstacles

6 - Simulation results with obstacles

6.1. Environment 2D maps

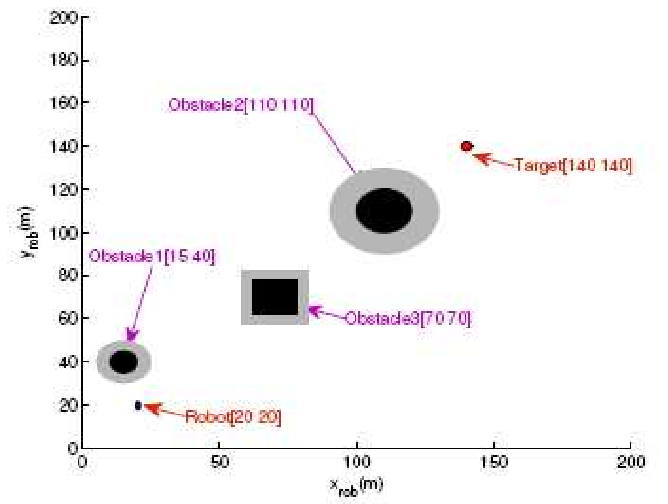
Parameters

m_{rob} is equal to [110, 150, 190, 250, 400]

the nominal mass is 150 kg

α_p=0.002 and α_v= 0.8 (Ge and Cui parameters)

fractional order n = 0.7



2D environment map

Initial conditions of each element

Element	*Position*	*Velocity*
Robot	[20,20]	[0,0]
Target	[140,140]	[0,0]
Obstacle 1	[15,40]	[0.01,0]
Obstacle 2	[110,110]	[0.01,-0.01]
Obstacle 3	[70,70]	[0,0]

Obstacles and replusive potential parameters

Repulsive potential parameters	ν
Obstacle 1	2.5
Obstacle 2	1.5
Obstacle 3	3

Attractive potential parameters

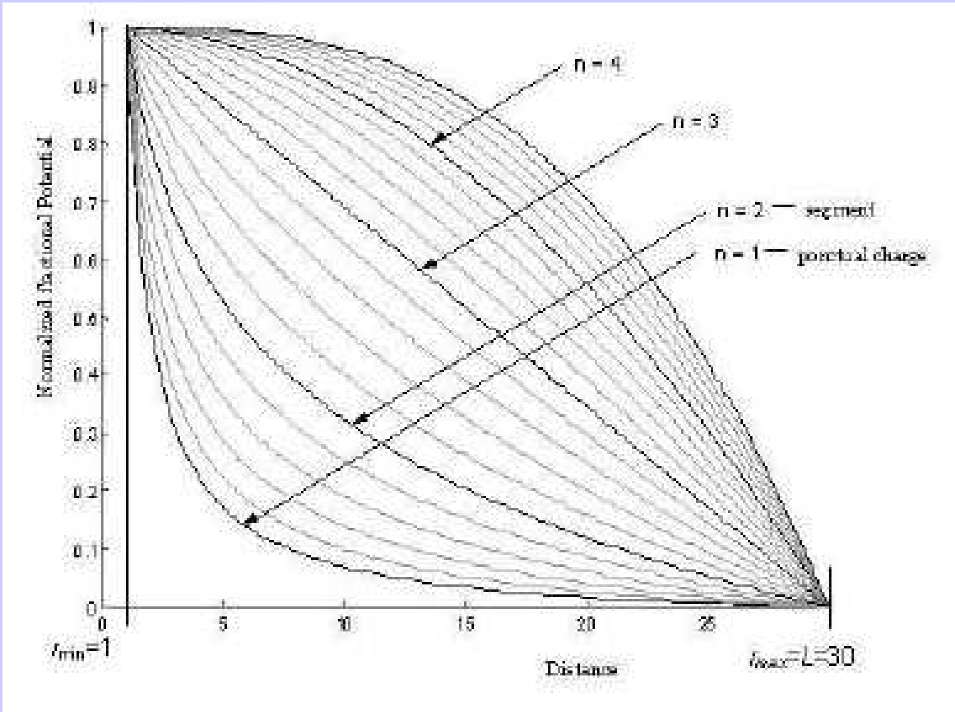
Attractive force parameters	α_p	α_v	n
Ge et Cui	0.002	0.8	-
Fractional	0.002	0.8	0.7

6 - Simulation results with obstacles

6.2. Fractional repulsive potential and danger

To characterize the obstacles and its danger, a ***fractional repulsive potential*** is used (Weyl's normalized fractional potential)

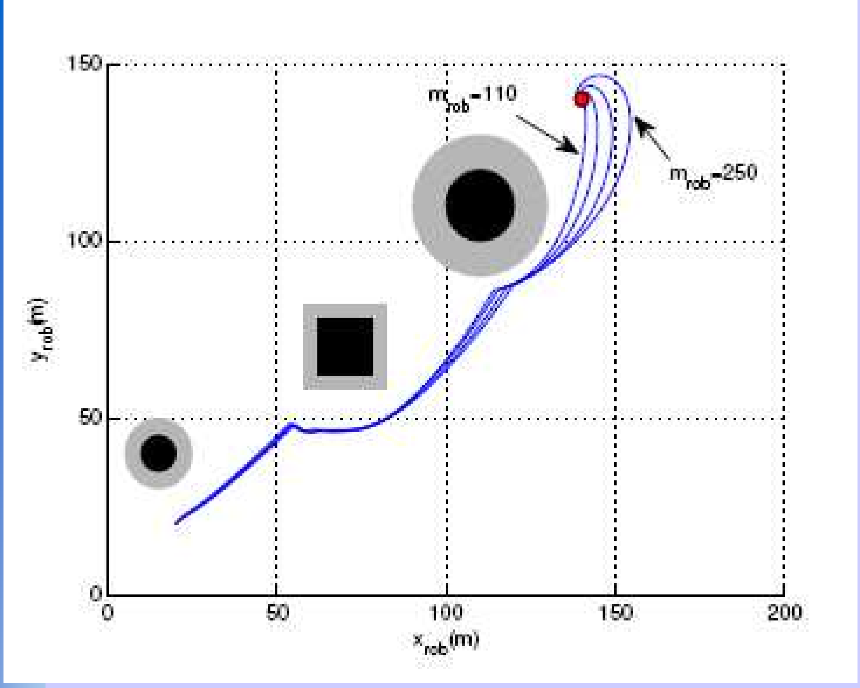
$$\forall \nu \in [0,2[\,,\ \forall r \in [r_{\min}, r_{\max}],\ U_{\nu}(r) = \frac{r^{\nu-2} - r_{\max}^{\nu-2}}{r_{\min}^{\nu-2} - r_{\max}^{\nu-2}}$$



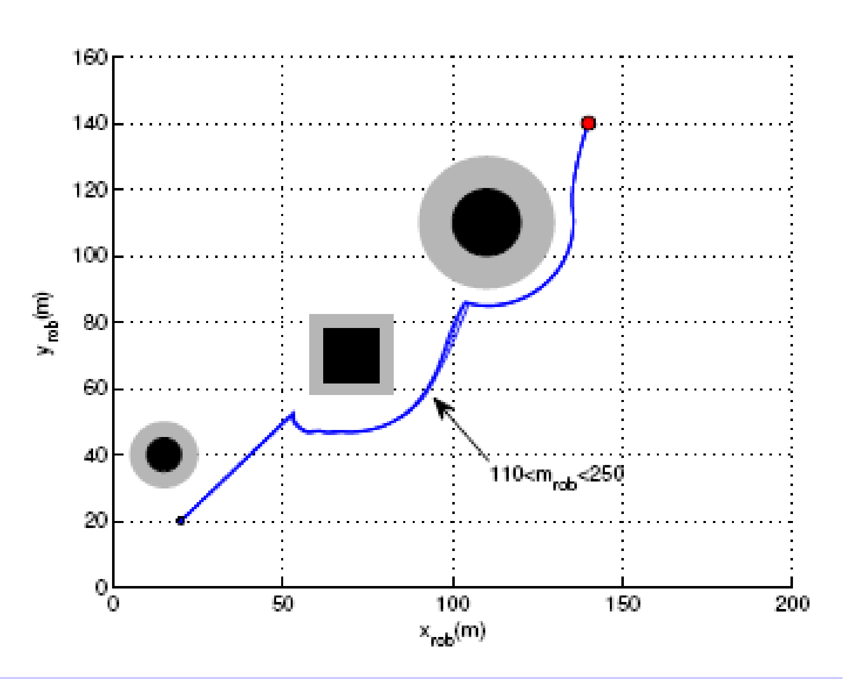
6 - Simulation results with obstacles

6.3. Simulation results in 2D static environment

Static target, static obstacles



Ge and Cui

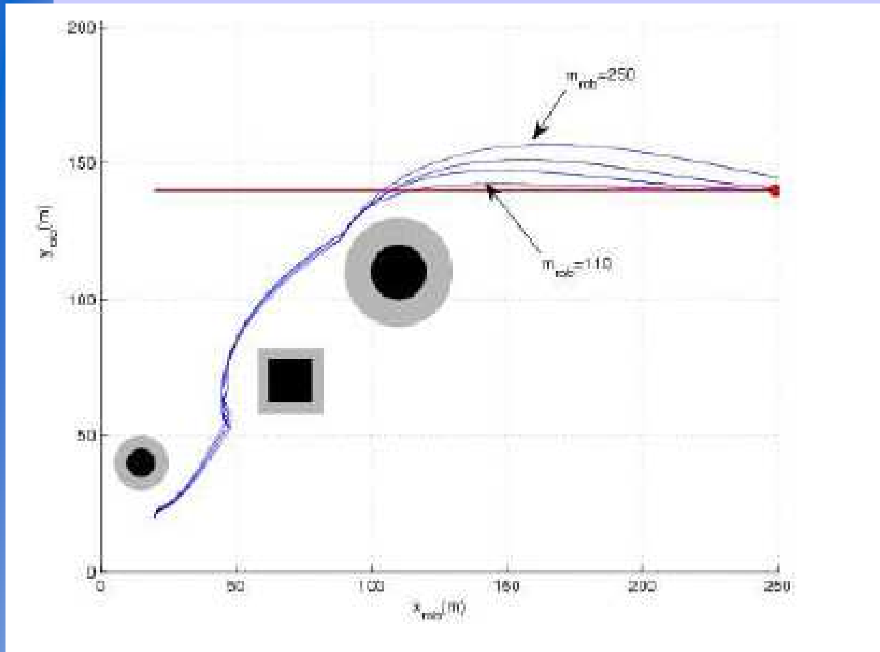


Fractional with same α_p, α_v

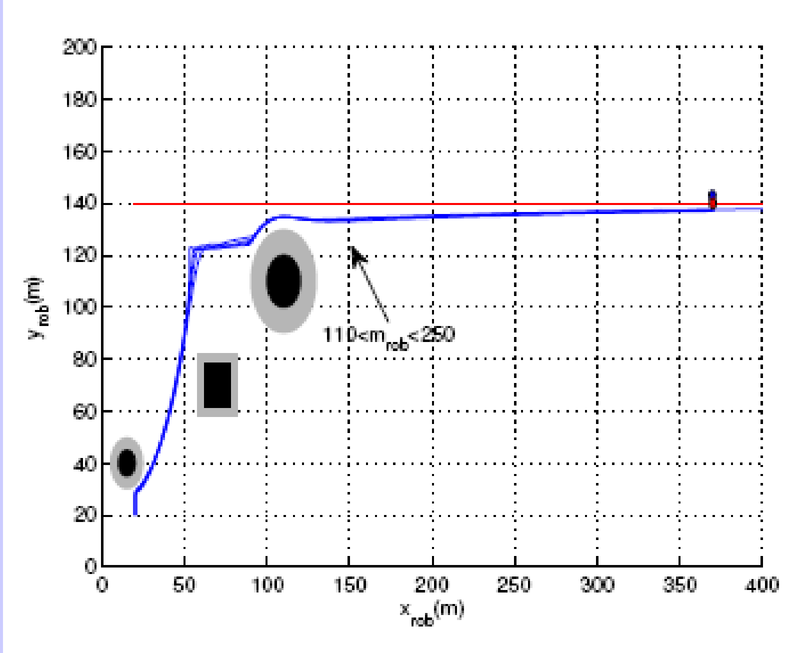
6 - Simulation results with obstacles

6.3. Simulation results in 2D static environment

Mobile target, static obstacles



Ge and Cui



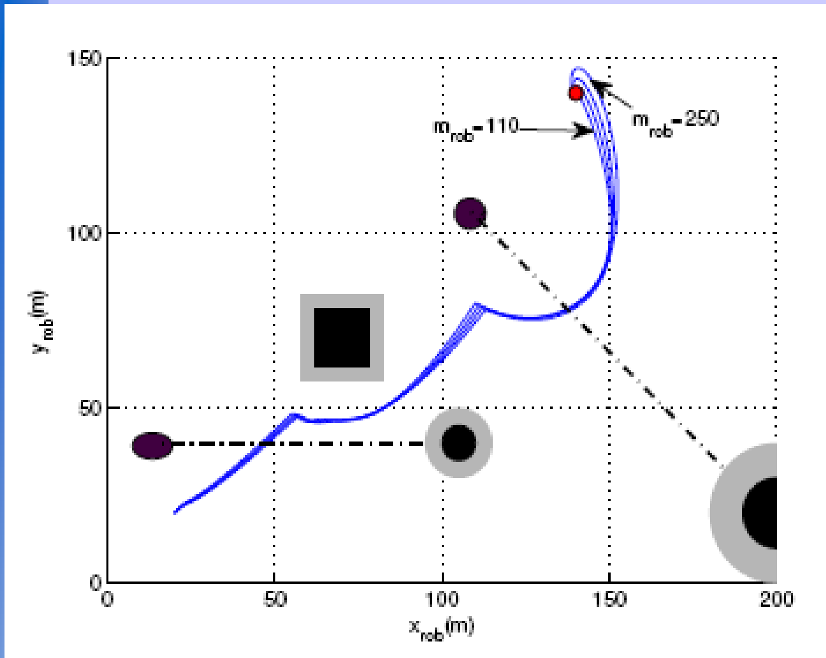
Fractional with same α_p, α_v

Robust and faster path planning despite robot mass variation

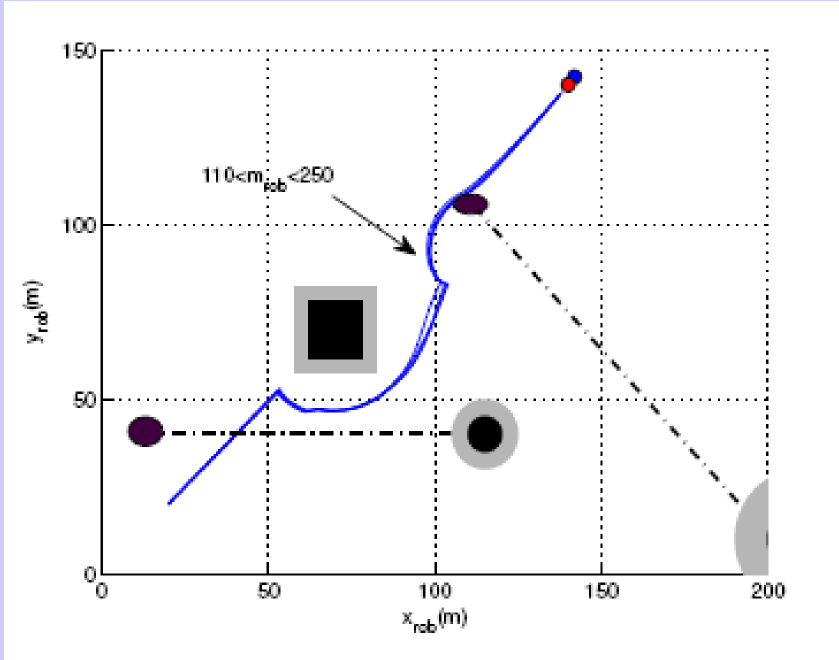
6 - Simulation results with obstacles

6.3. Simulation results in 2D dynamic environment

Static target, 1 static and 2 mobile obstacles



Ge and Cui



Fractional with same α_p, α_v

Robust and faster path planning despite robot mass variation

7 - Conclusion

Conclusion and prospects

In this paper:

- A new fractional attractive force for robust path planning of mobile robot in dynamic environment is presented.
- Robustness analysis
- This method allows to obtain robust and faster path planning despite robot mass variations.
- Simulations with static target and static and mobile obstacles

Future works

- To use this attractive force with mobile target and obstacles, and in 3D
- To take into account of the robot dynamic model
- Implementaion on a mobile robot and UAV