Toward a Unified Approach to the Control of Aerial Vehicles.

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Thrust-propelled underactuated vehicles

- Thrust force along a direction linked to the vehicle's main body
- Complete orientation actuation

Examples :

1. In the plane (SE(2)): boats, hovercrafts, (blimps, PVTOL)
2. In 3D space (SE(3)): submarines, aeroplanes, blimps, rockets, VTOL vehicles (helicopters, Xflyers, Bertin's HoverEye,...)

Objective :

develop the basics of a unified approach to the control of these systems

Core issues

- Nonlinear (vs. linear) control design
- Aerodynamic forces –attitude dependent lift, in particular– taken into account explicitly
- Extended operating domain (flight envelope), from hovering to high-velocity cruising, and possibly large angles of attack
- Application to a large variety of flying devices, including VTOL vehicles, airplanes, and convertible vehicles with transitions from hovering to cruising

Motion equations (Newton-Euler) 1/2

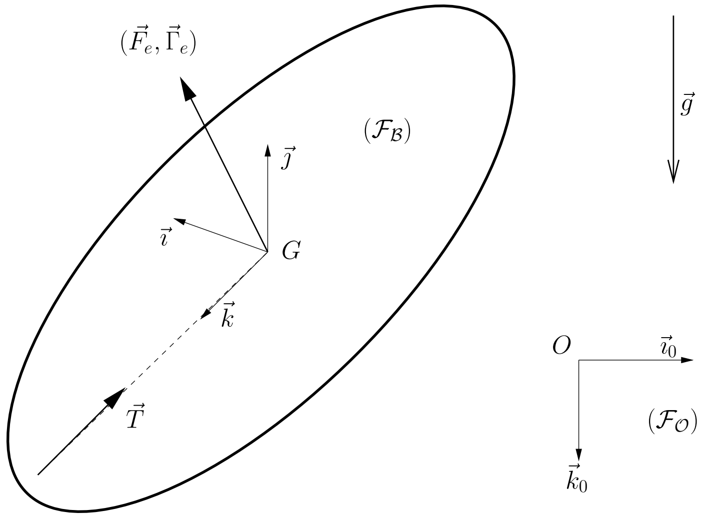
$\vec{a}$	:	vehicle's center of mass (c.o.m.) longitudinal acceleration
$\vec{g}$	:	gravity acceleration
$\vec{v}_a = \vec{v} - \vec{v}_{air}$	:	vehicle's longitudinal velocity relatively to ambient air
m	:	vehicle's mass
R	:	vehicle's orientation
$\vec{F}_a(\vec{v}_a, R)$	:	resultant of aerodynamic forces applied to the vehicle
$\vec{k}(R)$	:	unit vector opposite to the body-linked thrust force direction
T	:	thrust force intensity

$\vec{F}_e(\vec{v}_a, R) = \vec{F}_a(\vec{v}_a, R) + m\vec{g}$: resultant of external forces applied to the vehicle

$$\boxed{m\vec{a} = \vec{F}_e - T\vec{k}}$$: longitudinal motion

$$\vec{a} = 0 \;\Rightarrow\; T = |\vec{F}_e| \text{ and } \vec{k} = \frac{\vec{F}_e}{|\vec{F}_e|}$$

Motion equations (Newton-Euler) 2/2



R : rotation matrix between fixed-frame and body-fra

G : center of mass (c.o.m.)

$\frac{d}{dt}\vec{OG} = \vec{v} = (\vec{\imath}\,\vec{\jmath}\,\vec{k})v$

$\vec{\omega} = (\vec{\imath}\,\vec{\jmath}\,\vec{k})\omega$

Γ : actuation torque (vector of coordinates in body-fr

Γ_e : torque produced by external forces

$S(.)$: skew-symmetric matrix associated w.r.t. cross product in $\mathbb{R}^3$, i.e. $x \times y = S(x)y$

$$\begin{aligned} \dot{R} &= RS(\omega) \\ J\dot{\omega} &= -S(\omega)J\omega + \Gamma_e + \Gamma \end{aligned}$$

: rotation motion

Control problems from joystick augmentation to automatic flight

1. **Thrust direction** control (joystick augmentation mode, level 1)
2. **Velocity** control (joystick augmentation mode, level 2)
3. **Position** control, trajectory tracking (automatic flight)

For control purposes, minimal parametrizations of orientations (like Euler angles) should be banished. Use rotation matrices or quaternions.

Simplifying assumptions

1. The torque produced by $\vec{T}$ is small (control decoupling assumption)
2. Γ_e can always be "dominated " by the control torque Γ so that ω can be used as an intermediary control variable (backstepping technique)

(for problems 2 and 3 :)

3. $\vec{F}_e$ does not depends on the vehicle's orientation
   - example: the vehicle's shape is a sphere (helicopters)
   - counter-example: airplanes with large planar surfaces producing attack-angle dependent lift forces
4. $||\vec{F}_e(\vec{v}_a, R)|| \leq c_1 + c_2||\vec{v}_a||^2$
5. $\vec{v}_a.\vec{F}_e(\vec{v}_a, R) \leq c_3||\vec{v}_a|| - c_4||\vec{v}_a||^3$ (passivity property)
6. Complete measurement of the vehicle's state (position, orientation, velocities)
7. $\vec{F}_e$ is either measured or estimated
8. No actuation limitations in sign and amplitude

Thrust direction control

- $\vec{\gamma}(t) = (\vec{\imath}\,\vec{\jmath}\,\vec{k})\gamma(t) = (\vec{\imath}_0\;\vec{\jmath}_0\;\vec{k}_0)\gamma^{\mathcal{I}}(t)$: unit vector giving the desired thrust direction at time t
- Objective : exponential stabilization of $\vec{\gamma} - \vec{k} = \vec{0}$ ($\Leftrightarrow$ exp. stab. of $\theta = 0$ with $\cos(\theta) = \gamma_3$)

System :

$$\frac{d}{dt}\vec{k} = \vec{\omega} \times \vec{k}$$

Control :

$$\begin{cases} \omega_1 = -\frac{\beta\gamma_2}{(1+\gamma_3)^2} - {\gamma^{\mathcal{I}}}^T S(Re_1)\dot{\gamma}^{\mathcal{I}} \\ \omega_2 = -\frac{\beta\gamma_1}{(1+\gamma_3)^2} - {\gamma^{\mathcal{I}}}^T S(Re_2)\dot{\gamma}^{\mathcal{I}} \end{cases} \quad \beta > 0,\ e_1 = (1,0,0)^T,\ e_2 = (0,1,0)^T$$

$\omega_3(t)$ is free and the domain of attraction is $(-\pi, \pi)$.

Velocity control 1/2

- $\vec{v}_r$: desired velocity of G
- $\vec{a}_r = \frac{d}{dt}\vec{v}_r$: desired acceleration
- $\vec{\tilde{v}} := \vec{v} - \vec{v}_r$: velocity error $= (\vec{\imath}\,\vec{\jmath}\,\vec{k})\tilde{v} = (\vec{\imath}_0\ \vec{\jmath}_0\ \vec{k}_0)\tilde{v}^{\mathcal{I}}$
- $\vec{\gamma} := \frac{\vec{F}_e}{m} - \vec{a}_r$ $= (\vec{\imath}\,\vec{\jmath}\,\vec{k})\gamma = (\vec{\imath}_0\ \vec{\jmath}_0\ \vec{k}_0)\gamma^{\mathcal{I}}$

Error system

$$\begin{cases} \frac{d}{dt}\vec{\tilde{v}} = \vec{a} - \vec{a}_r = \vec{\gamma} - \frac{T}{m}\vec{k} \\ \frac{d}{dt}\vec{k} = \vec{\omega} \times \vec{k} \end{cases}$$

If $\|\vec{\gamma}\| > \epsilon > 0$, the asymptotic stabilization of $\vec{\tilde{v}} = 0$ is equivalent to the asymptotic stabilization of $(\|\vec{\gamma}\| - \frac{T}{m} = 0\ ,\ \frac{\vec{\gamma}}{\|\vec{\gamma}\|} - \vec{k} = \vec{0})$

- Existence of "classical" control solutions relies on the satisfaction of this assumption
- Ex: hovering VTOL vehicle ($\vec{v}_r = \vec{a}_r = \vec{0}$), no wind ($\vec{F}_a = \vec{0}$), submitted to gravity $\Rightarrow \vec{F}_e = m\vec{g} \neq \vec{0}$
- Counter-ex : Boat at rest ($\vec{v}_r = \vec{a}_r = \vec{0}$), no current + buoyancy $\Rightarrow \vec{F}_e = \vec{0}$

Velocity Control 2/2

Control 1 :

$$\begin{cases} \frac{T}{m} = \gamma_3 + \|\gamma\|\beta_1\tilde{v}_3 \\ \omega_1 = -\|\gamma\|\beta_2\tilde{v}_2 - \frac{\beta_3 \frac{\gamma_2}{\|\gamma\|}}{(1+\frac{\gamma_3}{\|\gamma\|})^2} - \frac{1}{\|\gamma\|^2}\gamma^{\mathcal{I}^T} S(Re_1) \dot{\gamma}^{\mathcal{I}} \\ \omega_2 = \|\gamma\|\beta_2\tilde{v}_1 - \frac{\beta_3 \frac{\gamma_1}{\|\gamma\|}}{(1+\frac{\gamma_3}{\|\gamma\|})^2} - \frac{1}{\|\gamma\|^2}\gamma^{\mathcal{I}^T} S(Re_2) \dot{\gamma}^{\mathcal{I}} \end{cases} \quad (\beta_{1,2,3} > 0)$$

Control 2 : with complementary integral action

Same control expression with $\vec{\gamma} := \frac{\vec{F}_e}{m} - \vec{a}_r + h(\|I_v\|^2)\vec{I}_v$

$I_v^{\mathcal{I}} = \int_0^t \tilde{v}^{\mathcal{I}}(s)ds + I_0^{\mathcal{I}}$

$h\ (.)$: positive function ensuring integral action boundedness

Ex: $h(s) = \frac{\eta}{\sqrt{(1+s)}}$

Position control - Trajectory tracking

- G_r : reference point in 3D-space
- $\vec{G_r G}$: position error $= (\vec{\imath}_0 \; \vec{\jmath}_0 \; \vec{k}_0)\tilde{x}^{\mathcal{I}}$

Control 1 : same as previous velocity control 2, since $\tilde{x}^{\mathcal{I}} = I_v^{\mathcal{I}}$ with $I_0^{\mathcal{I}} = 0$

Control 2 : incorporates a position integral term z, bounded and endowed with anti-windup properties. Yields same control expression with

$$\gamma := \frac{F_e}{m} - a_r + h(\|\tilde{x}^{\mathcal{I}} + z\|^2)(\tilde{x}^{\mathcal{I}} + z) + \ddot{z}$$

and $\tilde{v}$ replaced by $\bar{v} := \tilde{v} + R^T \dot{z}$

- Robustification adjustments in situations where $\|\gamma\|$ becomes small

When $\vec{F}_a$ depends on the vehicle's orientation 1/2

Case of a vehicle whose shape is symmetric about the thrust axis

- $\vec{F}_a = \vec{F}_D + \vec{F}_L$: decomposition into drag and lift forces
- $(\alpha = \cos^{-1}(-\frac{v_{a,3}}{|v_a|}), \beta = \text{atan2}(v_{a,2}, v_{a,1}))$: pair of angles characterizing the direction of $\vec{v}_a$ w.r.t. the vehicle's frame
- $\vec{r}(\beta) = -\sin(\beta)\vec{\imath} + \cos(\beta)\vec{\jmath}$
- (R_e, M) : Reynolds and Mach numbers

Combining *Buckingham π-theorem* and body symmetry yields :

$$\vec{F}_D = -k_a \|\vec{v}_a\| C_D(R_e, M, \alpha)\vec{v}_a$$
$$\vec{F}_L = \ k_a \|\vec{v}_a\| C_L(R_e, M, \alpha)\vec{r}(\beta) \times \vec{v}_a$$

with (C_D, C_L) : *aerodynamic characteristics* of the vehicle's body (independent of β)

When $\vec{F}_a$ depends on the vehicle's orientation 2/2

Transformation into the spherical case

If :

$$(1) \quad \forall \alpha \ : \ C_D(\alpha) + C_L(\alpha)\cot(\alpha) = C_{D_0}$$ (a constant number)

then :

$$\begin{aligned} m\vec{a} &= \vec{F}_a(\vec{v}_a, R) + m\vec{g} - T\vec{k} \\ &= \vec{F}_{as}(\vec{v}_a) + m\vec{g} - T_s\vec{k} \end{aligned}$$

with :

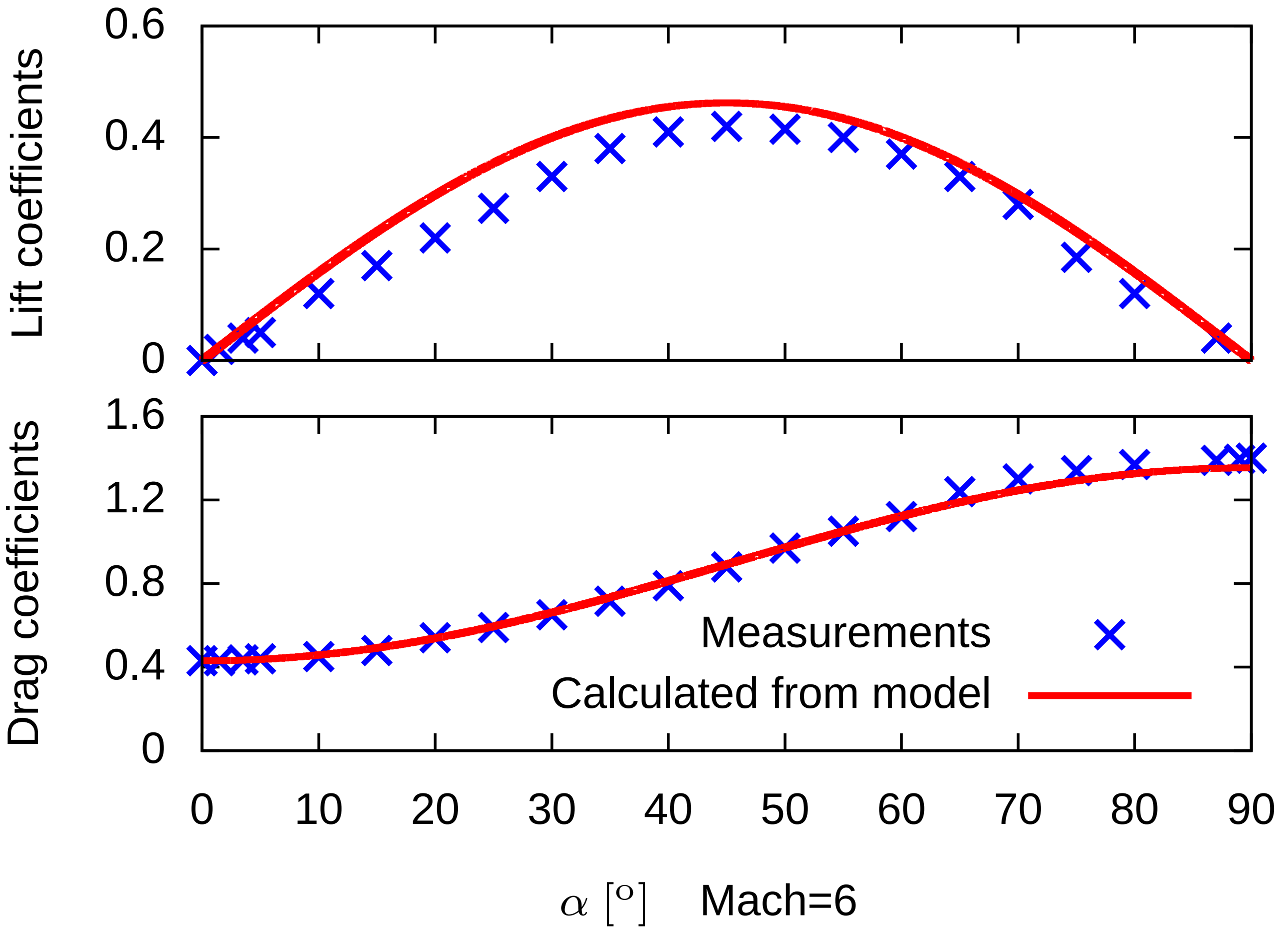
$\vec{F}_{as}(\vec{v}_a) = -k_a C_{D_0} \|\vec{v}_a\| \vec{v}_a$: independent of α (of the body's orientation)

$T_s = T + k_a \|\vec{v}_a\|^2 \frac{C_L(\alpha)}{\sin(\alpha)}$

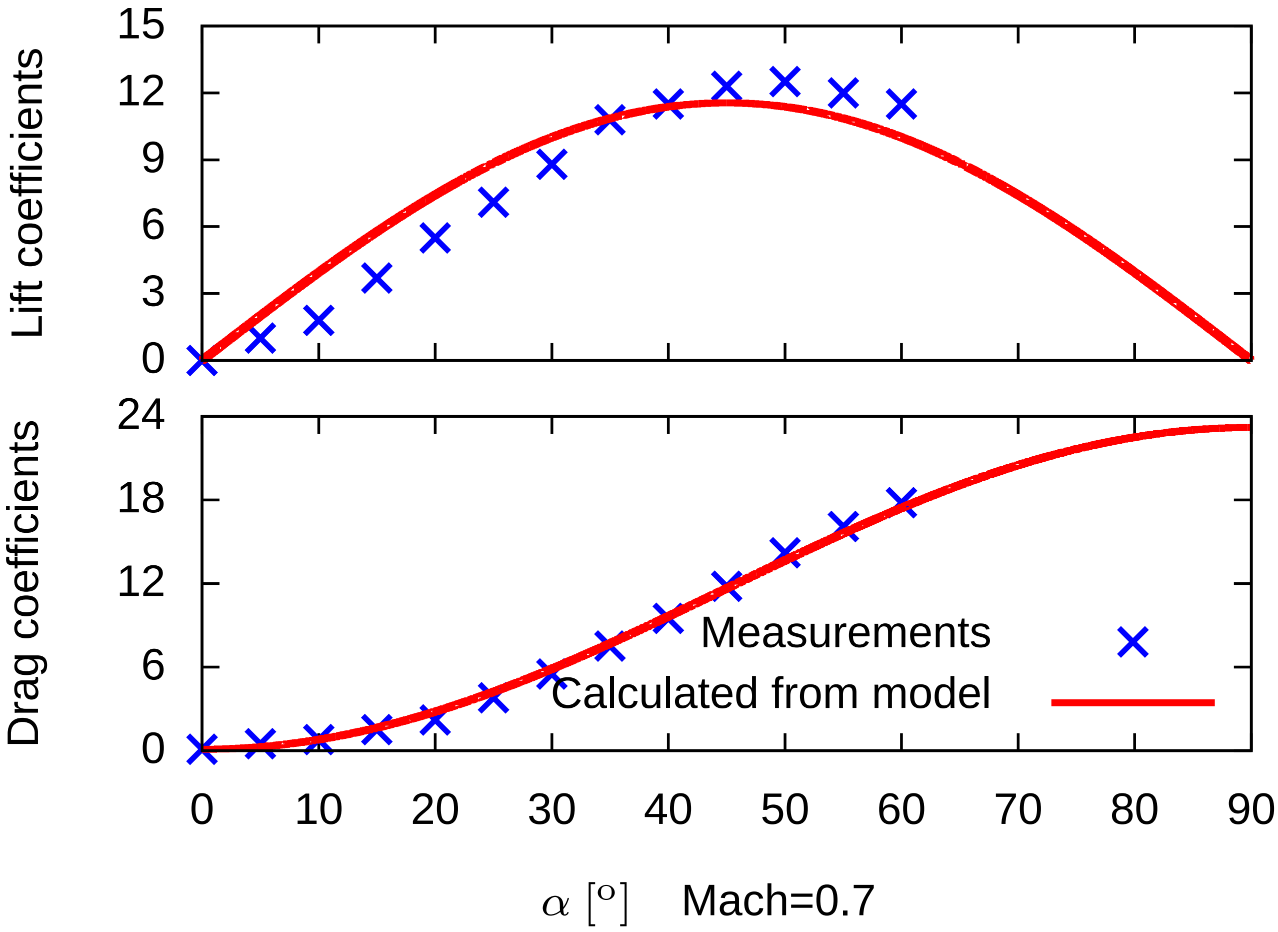
Examples of aerodynamic characteristics satisfying (1)

$$\begin{cases} C_D(\alpha) = c_0 + 2c_1(\sin(\alpha))^2 \\ C_L(\alpha) = c_1 \sin(2\alpha) \end{cases} \qquad \begin{cases} C_D(\alpha) = \bar{c}_0 \\ C_L(\alpha) = \bar{c}_1 \tan(\alpha) \end{cases}$$

Characteristics of an ellipsoidal body (Keyes, 1965)

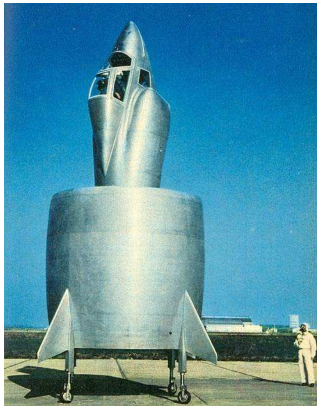


Characteristics of a missile (Saffel, Howard, Brooks, 1971)



Simulations

Spherical case (no aerodynamic lift, drag independent of vehicle's orientation)

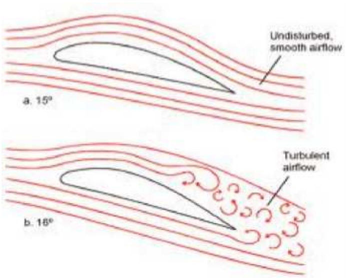


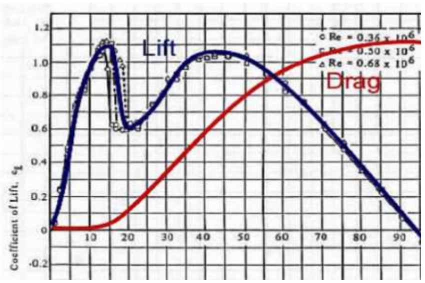
Annular wing (aerodynamic lift, symmetry about thrust direction)

Perspectives

- case of symmetry-breaking flat wings (common airplanes), and relation with slide-slip angle zeroing via roll (ailerons) monitoring

- stall phenomenon (modeling, consequences, avoidance)





- actuation specialization (and associated limitations)
- measurement, estimation, multisensory fusion issues
- convergence with classical (linear) control approach and solutions
- etc.